EPFL

Cours d'Electrotechnique I

2. Conventions et les symboles :

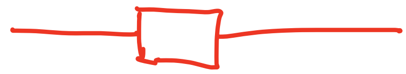
→ concepts → modèle

Ex : courant : i, I, $\hat{I}$, $\hat{i}$, $\bar{I}$, $\underline{I}$

unité : [A]

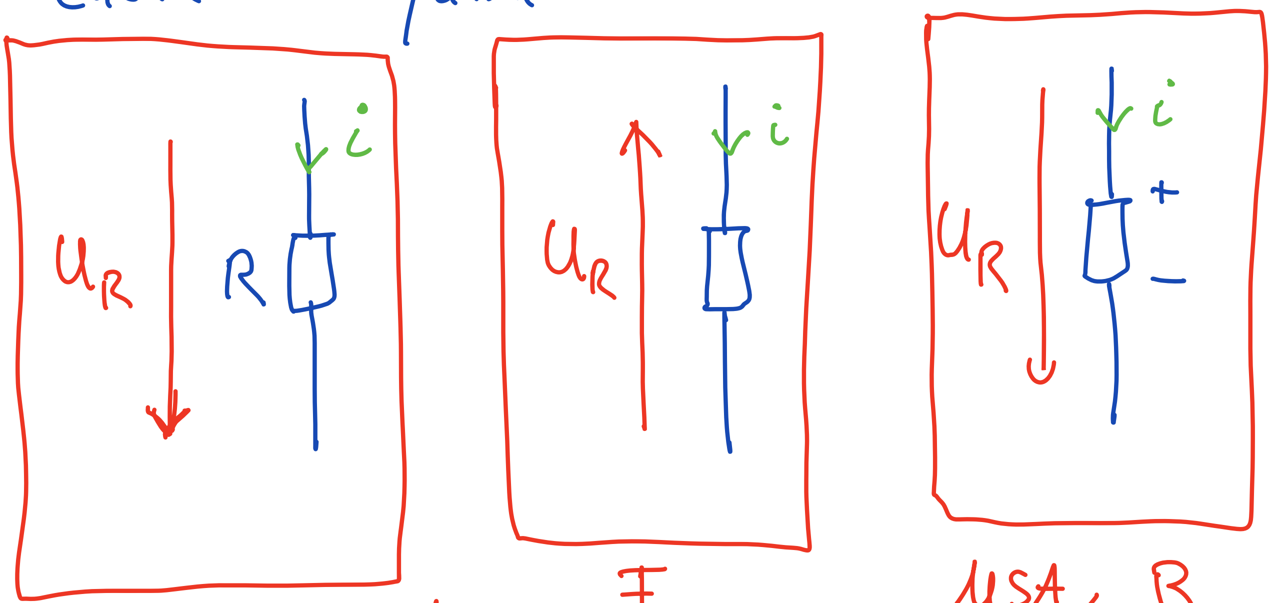
Relations : $U = R \cdot I$

$u = R \cdot i$

Dessin : 

Résistance

Choix à faire :

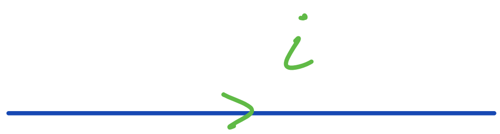


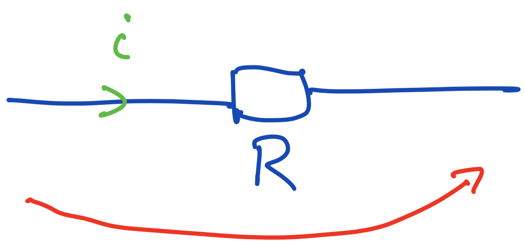
International — Fr — USA, B

convention moteur : choix

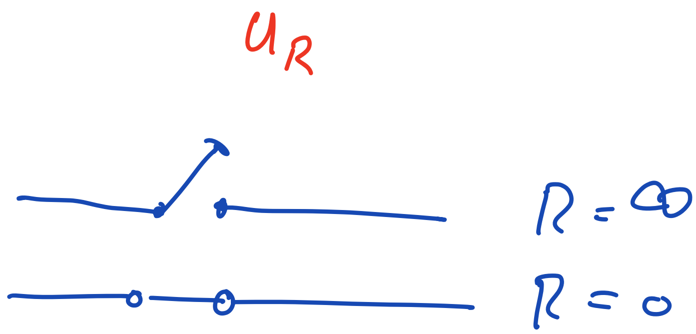
2.2 Représentation graphique :

Conducteur parfait :

conducteur avec un courant : 

Élément : 

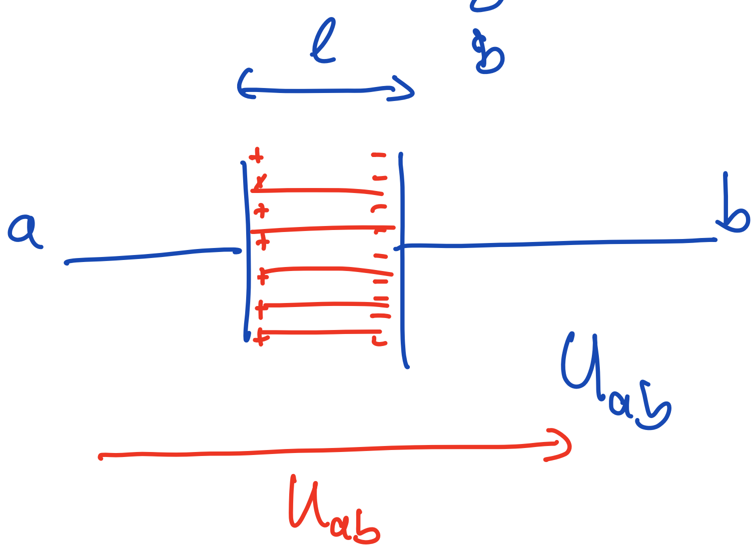
Interrupteur :



3. Lois fondamentales :

Différence de potentiel : Tension [Volt]

$$V_a - V_b = \int_b^a E\,dl = U_{ab} \quad [V]$$



3.2.19 La Capacité :

Définition : Charge électrique : Q

Capacité : $C = \dfrac{Q}{U_{ab}}$

Symbole : 

3.3 Courant électrique :

$$I = \frac{dQ}{dt} \quad [A]$$

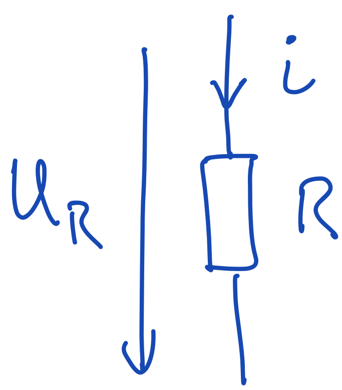
Densité de courant : $J \Rightarrow [A/m^2]$

3.3.4 Pertes Joule :

$$P = R \cdot I^2 \quad [W]$$

Récap :

Convention moteur :



→ Puissance positive

→ conv. moteur consommateur.

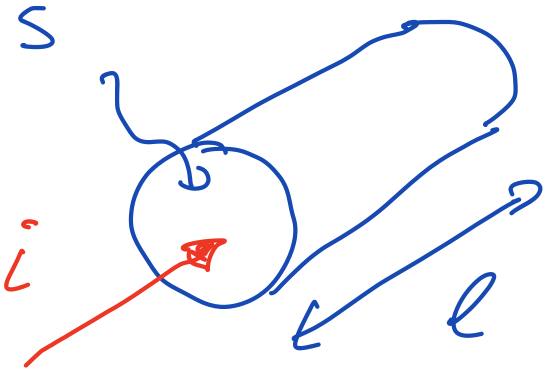
3.3.6 Définition de la résistance :

$$R_{ab} = \int_a^b \rho \cdot \frac{dl}{S}$$

longueur

surface

résistivité électrique $[\Omega m]$



Si S est constante sur la longueur

$$R_{ab} = \frac{\rho \cdot l}{S} \quad [\Omega]$$

3.3.8 Loi d'Ohm :

$$U_{ab} = R_{ab} \, I$$

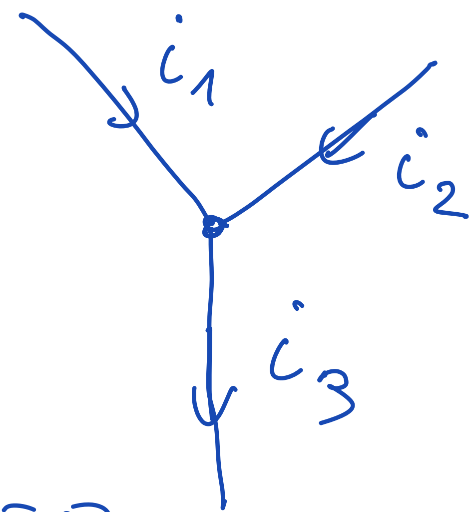
(courant et tension continue)

$u_{ab} = R_{ab} \cdot i$ (courant et tension variable)

3.3.11 Lois de Kirchhoff :

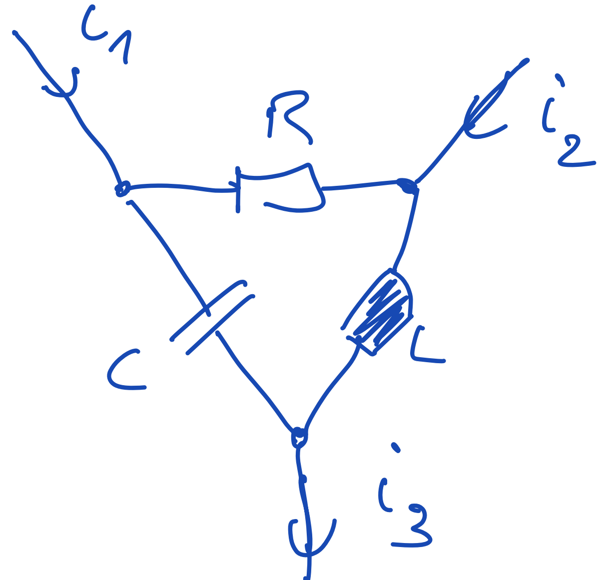
Noeud : Point de convergence d'au moins trois conducteurs

$$\sum i_j = 0$$



$i_1 + i_2 - i_3 = 0$

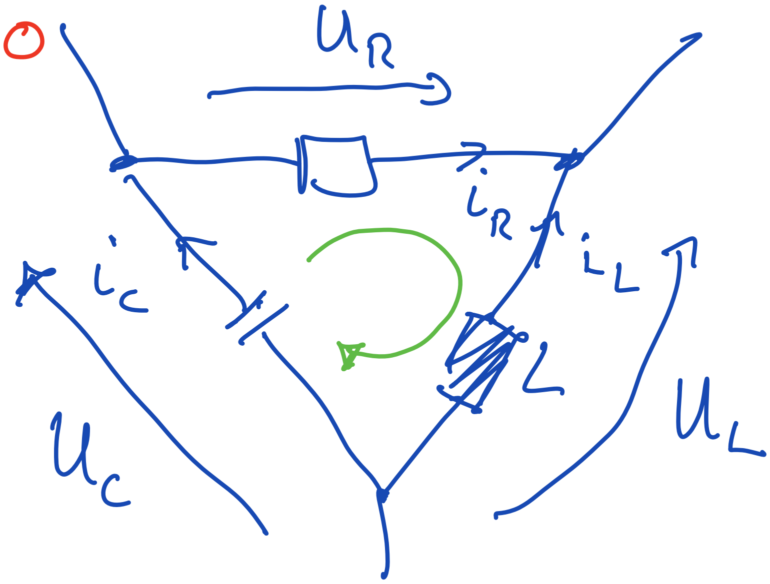
Noeud généralisé :



$i_1 + i_2 - i_3 = 0$

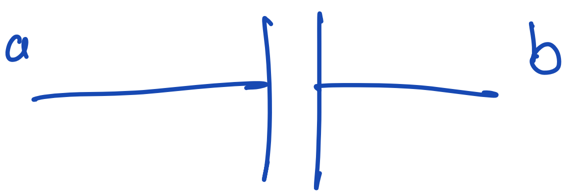
Maille : ensemble de branche partant d'un noeud pour y retourner

$$\sum U_j = 0$$



$$U_R - U_L + U_c = 0$$

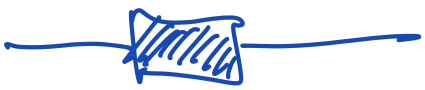
3.5 La Capacité :



$$C = \frac{Q}{U_{ab}} \qquad Q = \int i\,dt$$

$$u = \frac{1}{C}\int i\,dt$$

3.4 l'inductance :



L

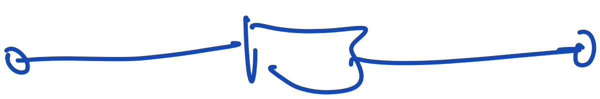
L

$$u = L\frac{di}{dt} \quad \begin{cases} \overrightarrow{\mathrm{Rot}}\,\vec{H} = \vec{J} \\ \overrightarrow{\mathrm{Rot}}\,\vec{E} = -\dfrac{d\vec{B}}{dt} \end{cases}$$

L [H] Henry

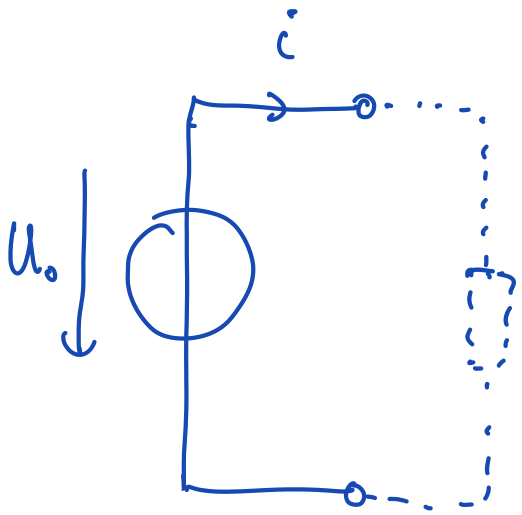
4. Eléments de circuit :

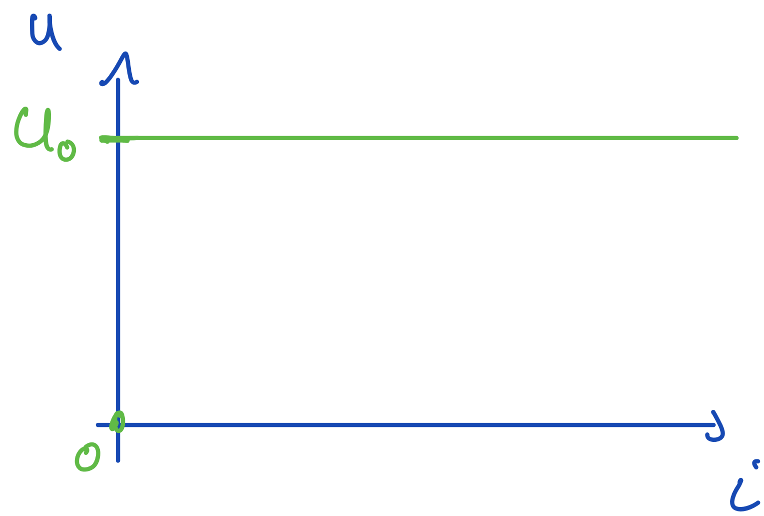
4.1 Dipôle : circuit qui possède 2 bornes



4.2 Sources de tension et de courant

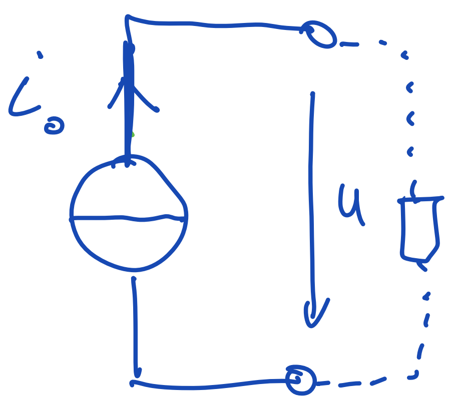
a) Source de tension idéale :

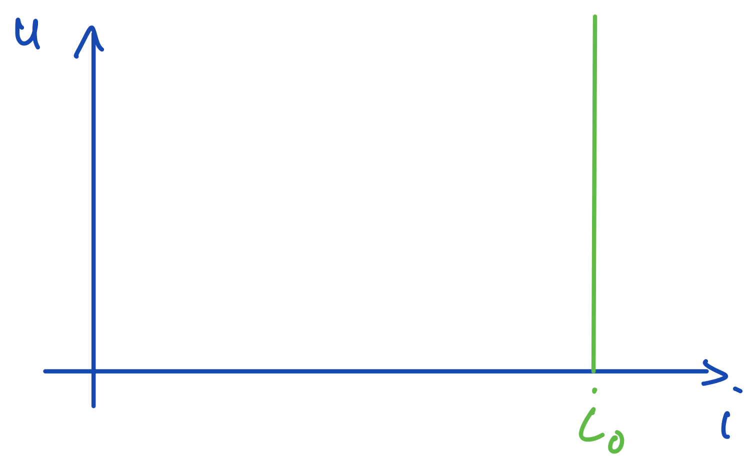




c'est un élément virtuel, idéal et inexistant dans la nature

b) Source de courant idéale :

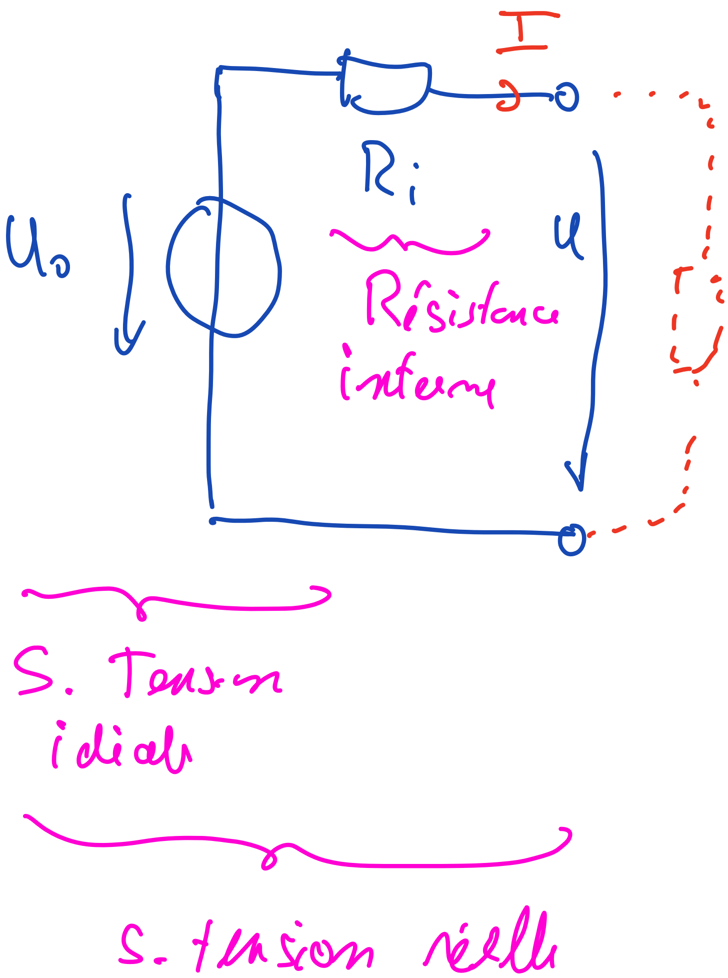




élément virtuel, inexistant dans la nature.

4.2.5 Source de tension réelle :

Déf :

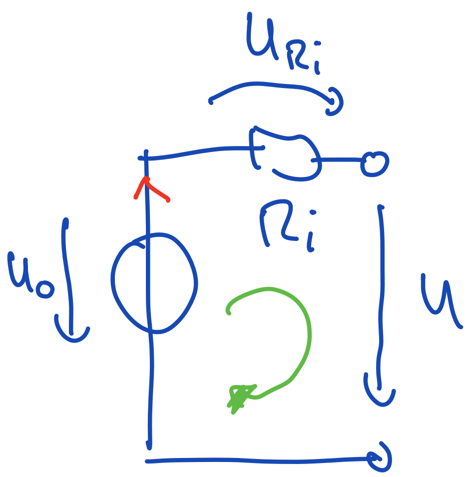


U_0 : Tension de la source idéale

Tension à vide

R_i : Résistance interne

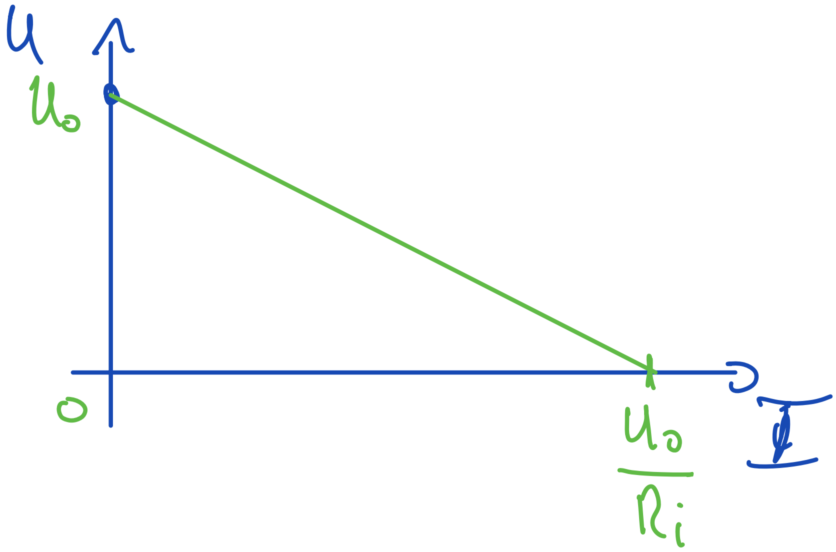
U : Tension de la source



$$\sum U = 0$$

$$-U_0 + \underbrace{U_{R_i}}_{= R_i \cdot I} + U = 0$$

$$U = U_0 - R_i \cdot I$$



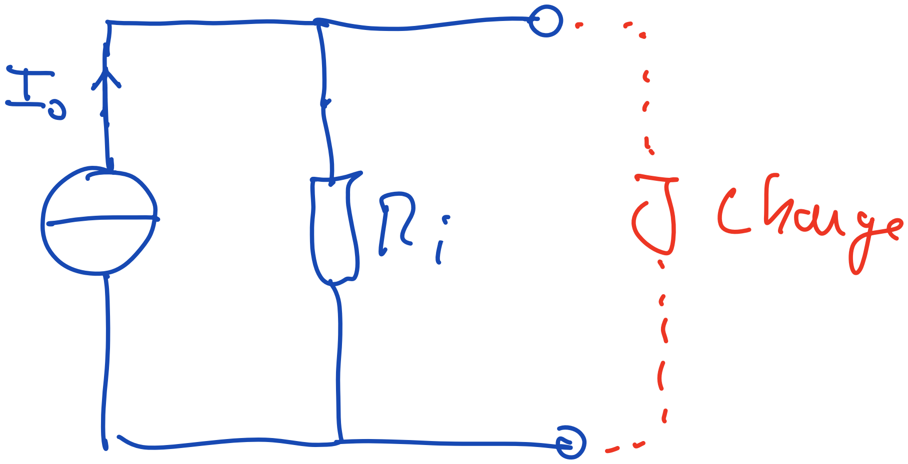
courant max :

$$U = 0$$

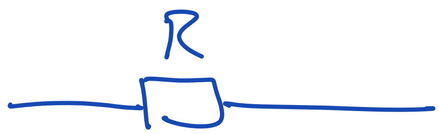
$$0 = U_0 - R_i \cdot I_{cc}$$

$$I_{cc} = \frac{U_0}{R_i}$$

4.2.6 Source de courant réelle :



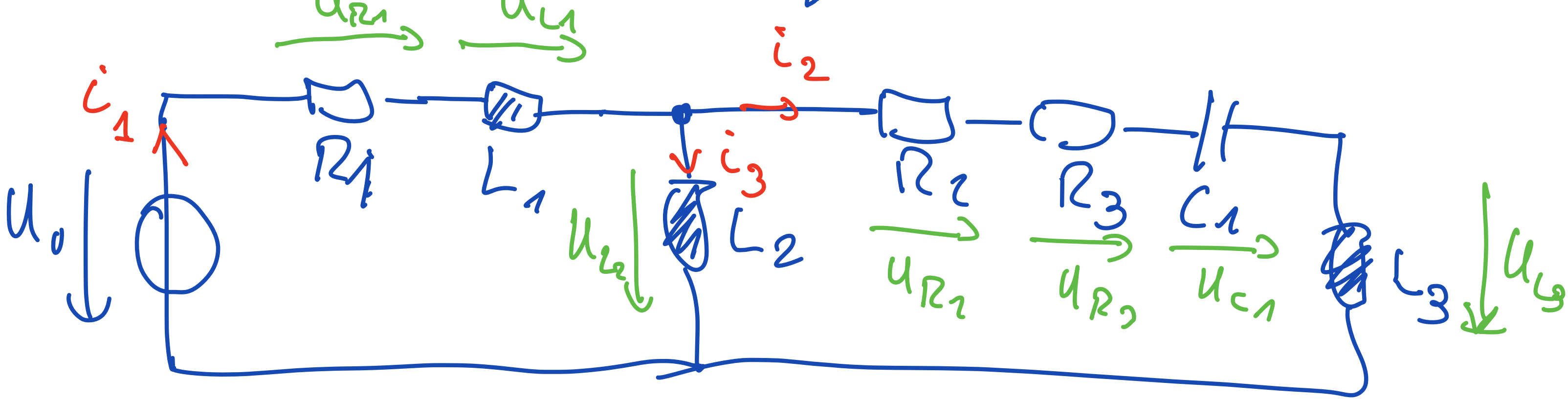
4.3 Elément de base :

Résistance 

Inductance 

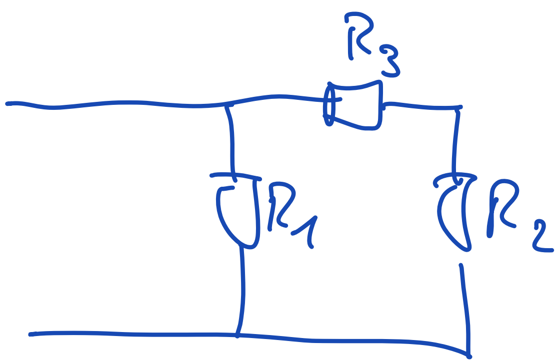
Capacité 

4.4 Schéma électrique :



Recap : Quiz :

4 : // → même tension aux bornes



R_1 n'est pas en // avec R_2 !

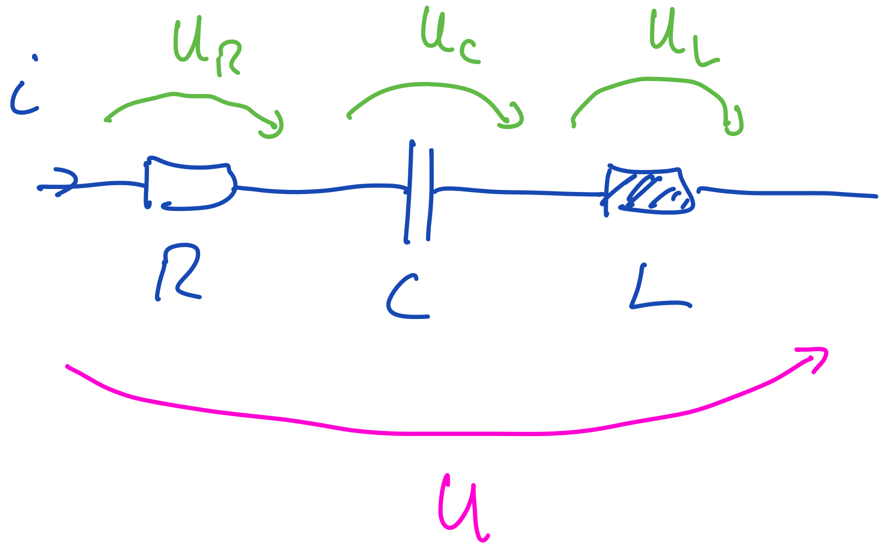
7 : Source idéale seule impossible.

8 : Source idéale est toujours constante

9 : Impossible de mettre des sources de courant en série.

5. Combinaison simple d'éléments linéaires

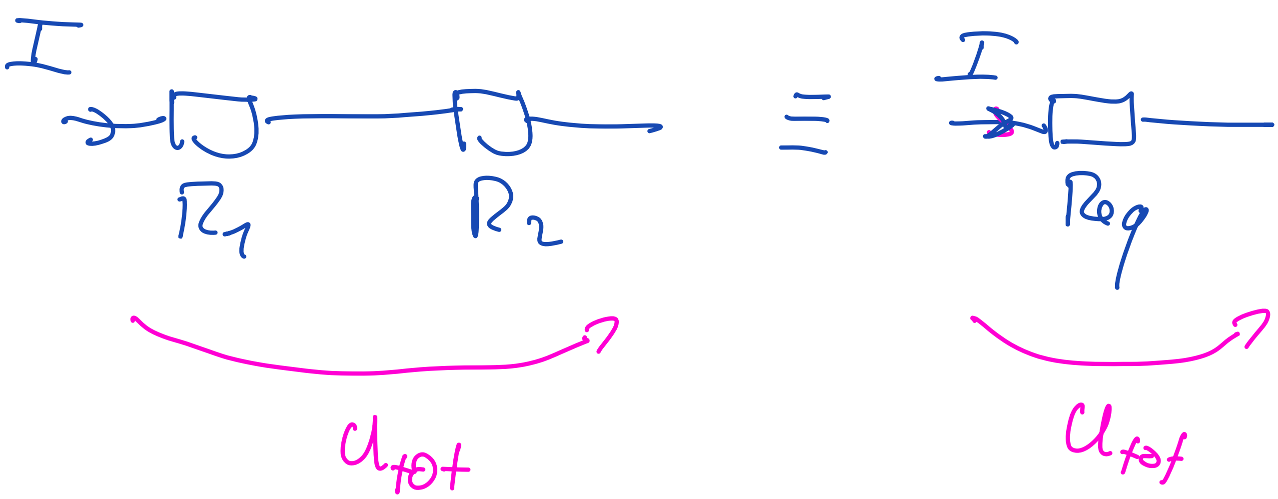
5.2 Mise en série :



Série : parcouru par le même courant

$$i_R = i_L = i_C \rightarrow \text{Série}$$

5.2.2 Mise en série de la résistance



$U_{tot} = U_{R_1} + U_{R_2}$ $\qquad$ $U_{tot} = R_{eq} \cdot I$

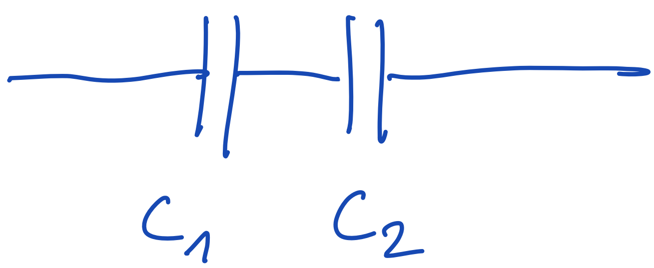
$= R_1 I + R_2 I$

$= (R_1 + R_2) I = R_{eq} \cdot I$

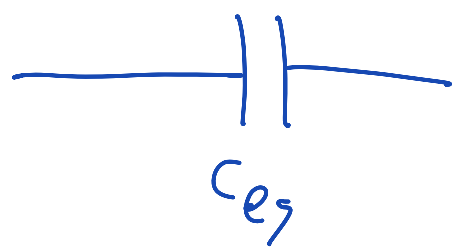
$\Longrightarrow R_{eq} = R_1 + R_2$

En série $R_{eq} = \sum_{K=1}^{n} R_K$ (n = nb de R)

5.2.3 Mise en série des C

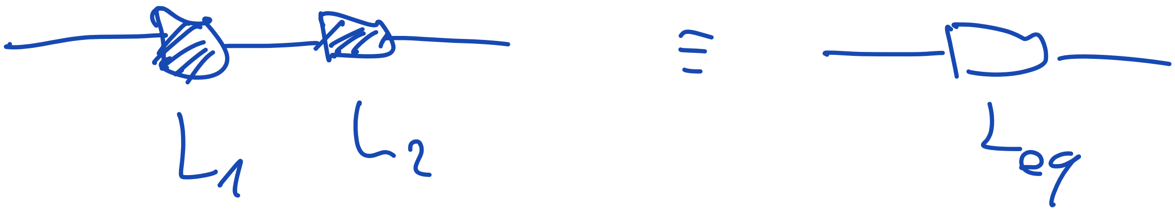


$\equiv$ $C_{eq} = ?$



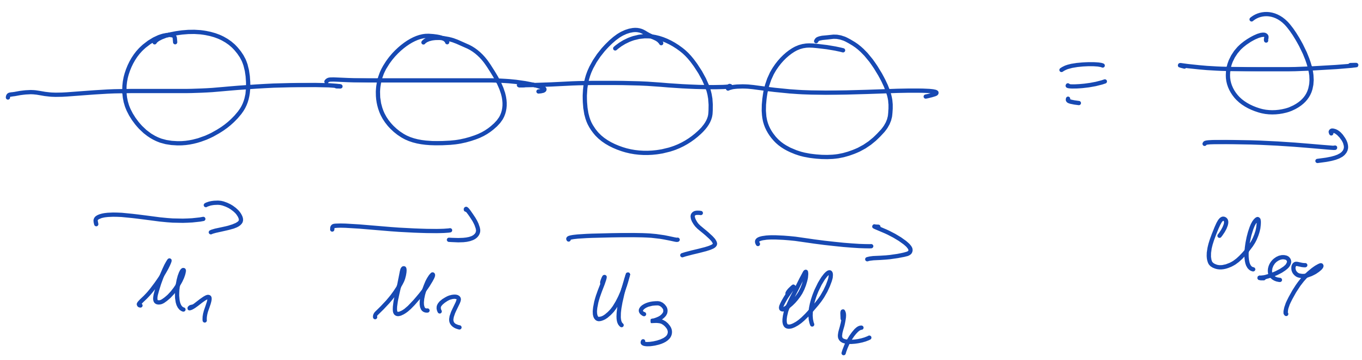
Série $C_{eq} = \dfrac{1}{\sum_{K=1}^{n} \dfrac{1}{C_K}}$ $\qquad$ n = nb de C

5.2.6 Mise en série des L

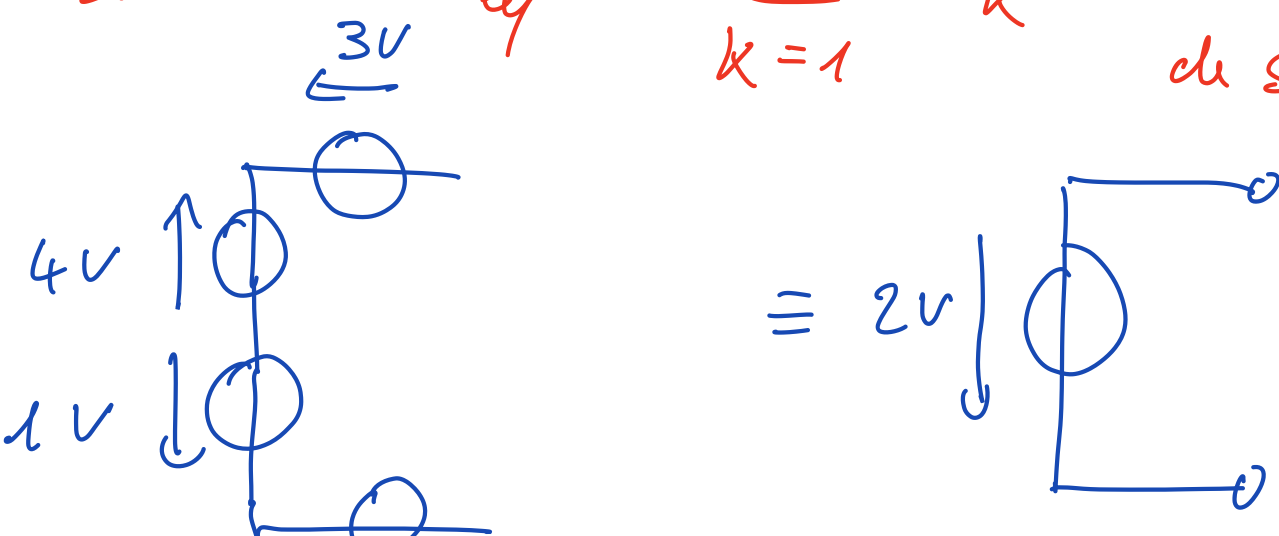


Série $L_{eq} = \sum_{k=1}^{n} L_k$ n = nb de L

5.2.7 Mise en série de source de tension

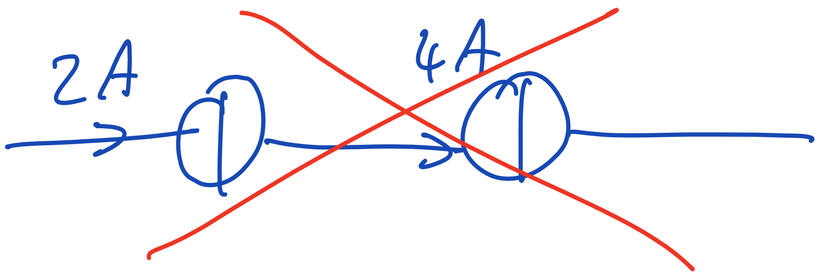


Série $U_{eq} = \sum_{k=1}^{n} U_k$ n = nb de source



$\overrightarrow{2U}$

5.2.9 Mise en série de source de courant

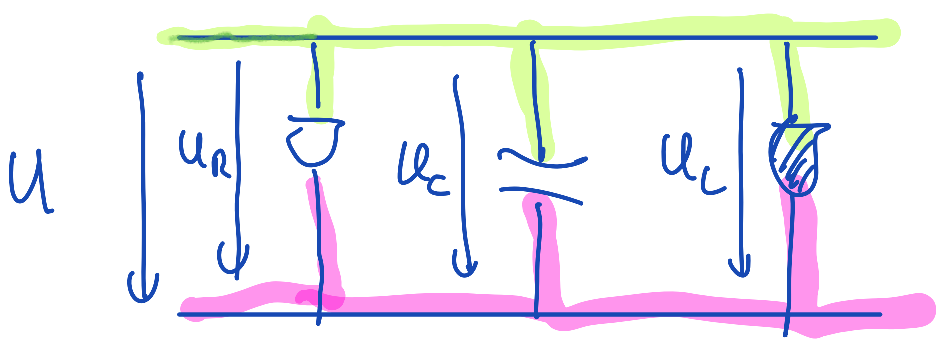


impossible

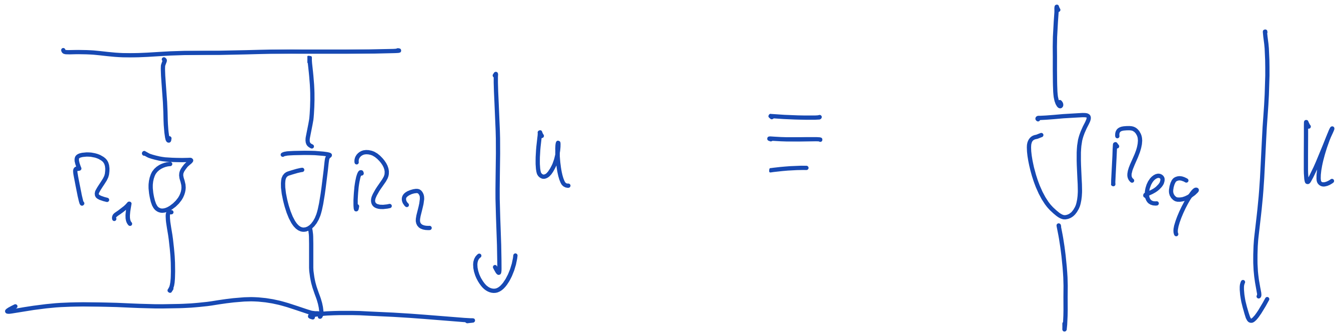
$\Rightarrow$ Impossible sauf si toutes les sources ont le même courant

5.3.2 Mise en // des R :

Définition : Toutes les bornes des éléments sont au même potentiel



$$U_R = U_C = U_L$$

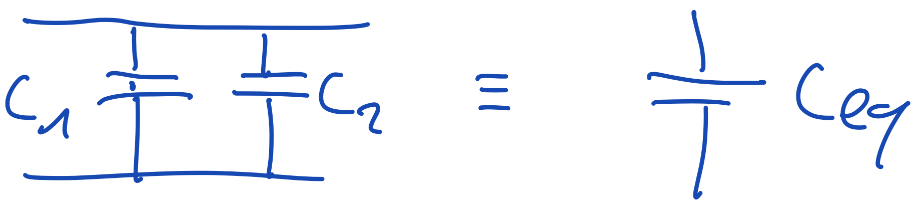


$$R_{eq} = \frac{1}{\sum_{k=1}^{m} \frac{1}{R_k}}$$

m = nb de R

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

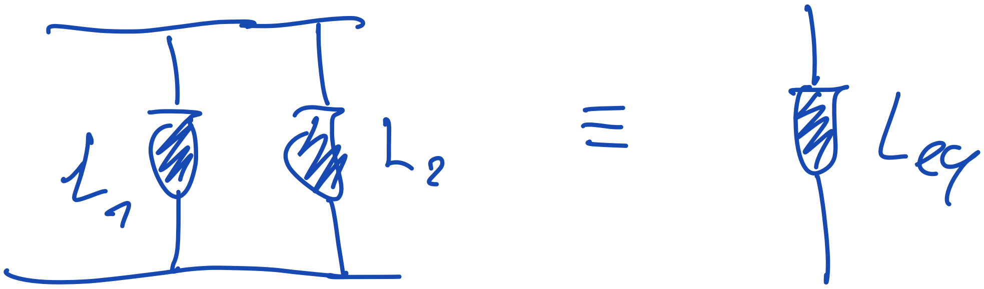
5.3.5 Mise en // des C :



$$// \quad C_{eq} = \sum_{k=1}^{m} C_k$$

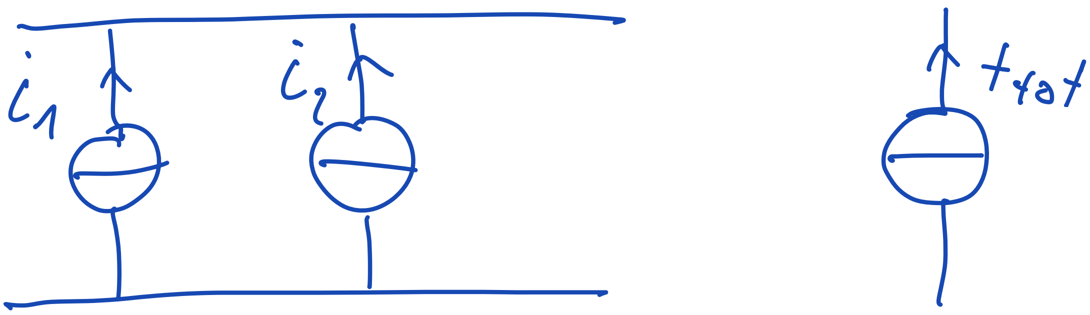
m = nb de C

5.3.6 Mise en // des L



$$// \quad L_{eq} = \frac{1}{\sum_{k=1}^{m} \frac{1}{L_k}} \qquad m = \text{nb de } L$$

5.3.7 Mise en // des sources de courant

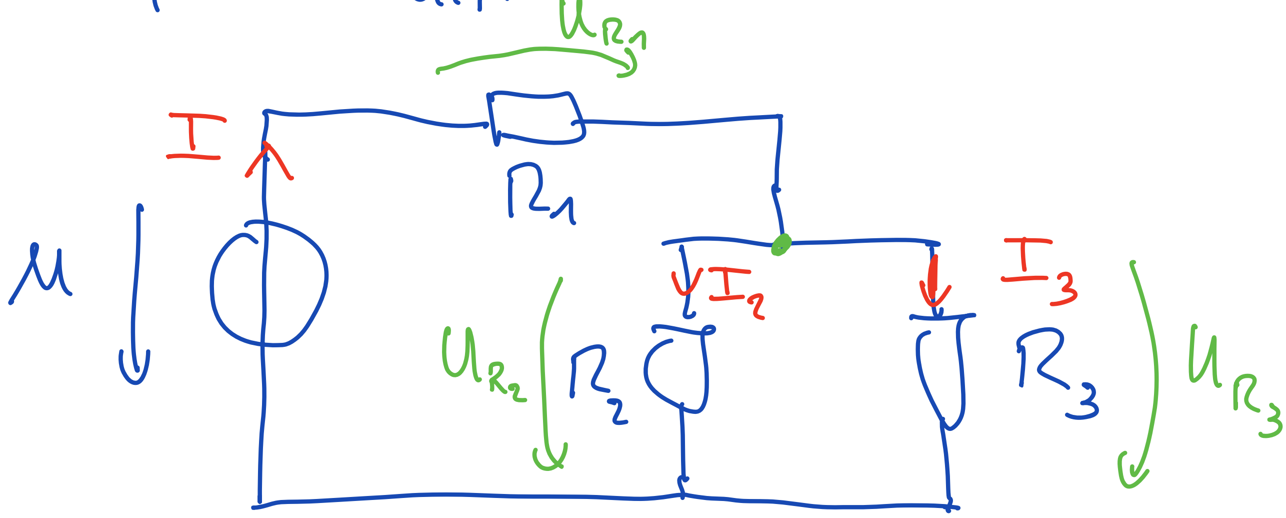


$$i_{tot} = \sum_{k=1}^{m} i_k$$

Mise en // de sources de tension

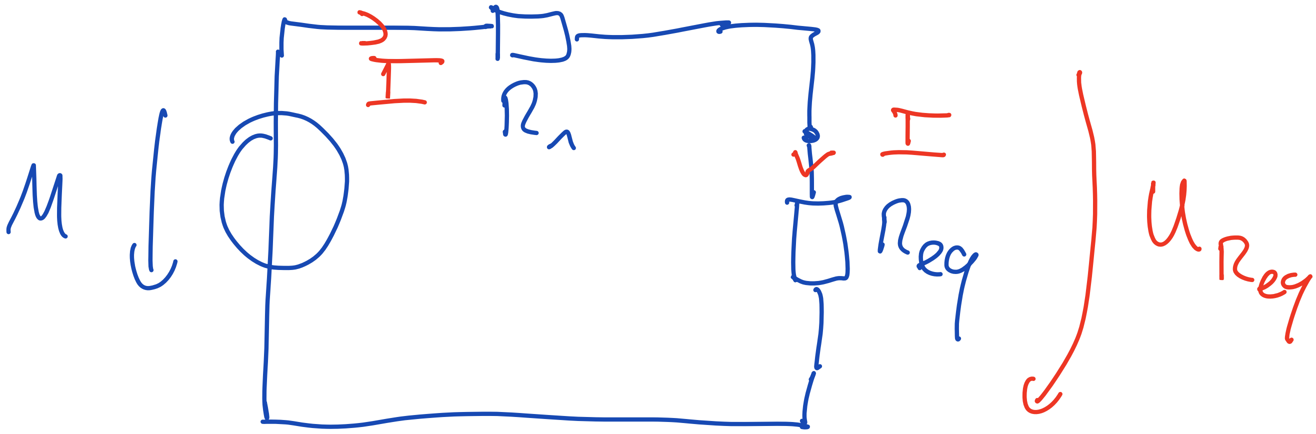
est impossible sauf si toutes les tension ont la même valeur

5.4 Circuits combinés :

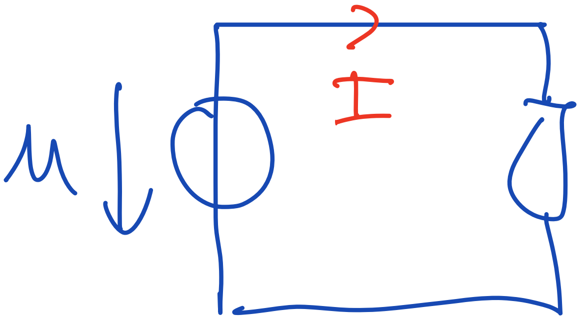


$I = ?$ $\quad I_2 ?$ $\quad I_3 ?$

$R_2 // R_3$



$$R_{eq} = \frac{R_2 \cdot R_3}{R_2 + R_3}$$



$$R_{tot} = R_1 + R_{eq}$$

$$= R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3}$$

$$U = R_{tot} \cdot I$$

$$I = \frac{U}{R_{tot}}$$

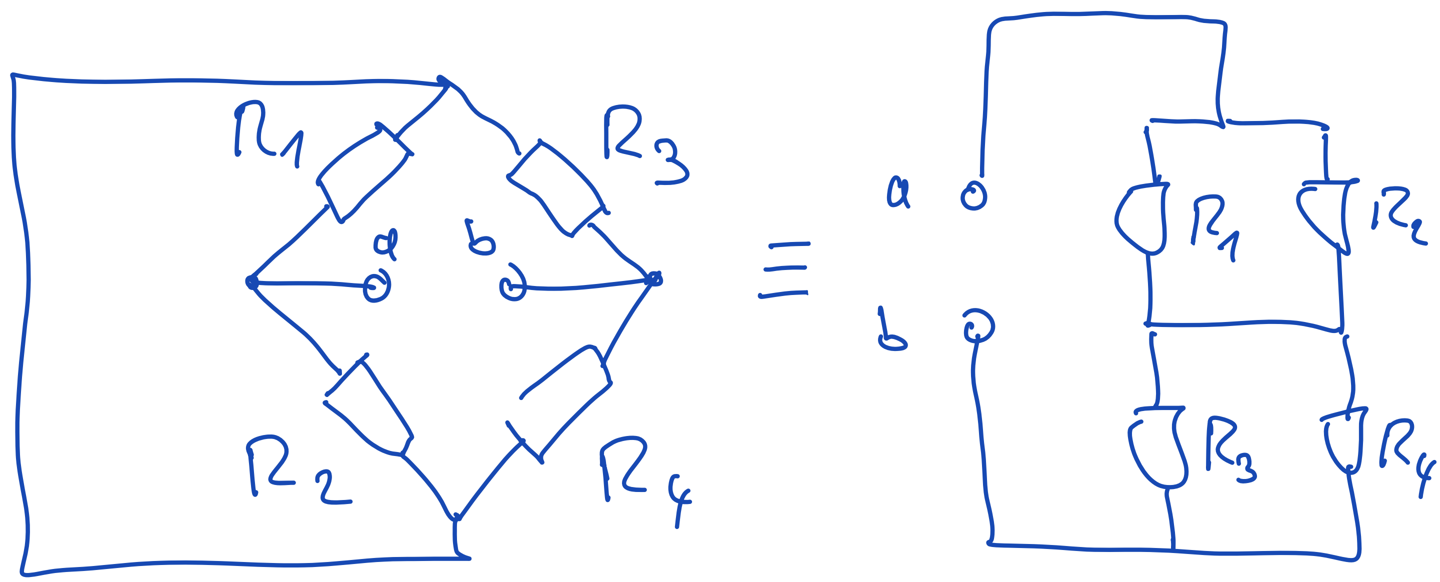
$$U_{R_2} = U_{R_3} = U_{R_{eq}} = R_{eq} \cdot I$$

$$= R_{eq} \cdot \frac{U}{R_{tot}}$$

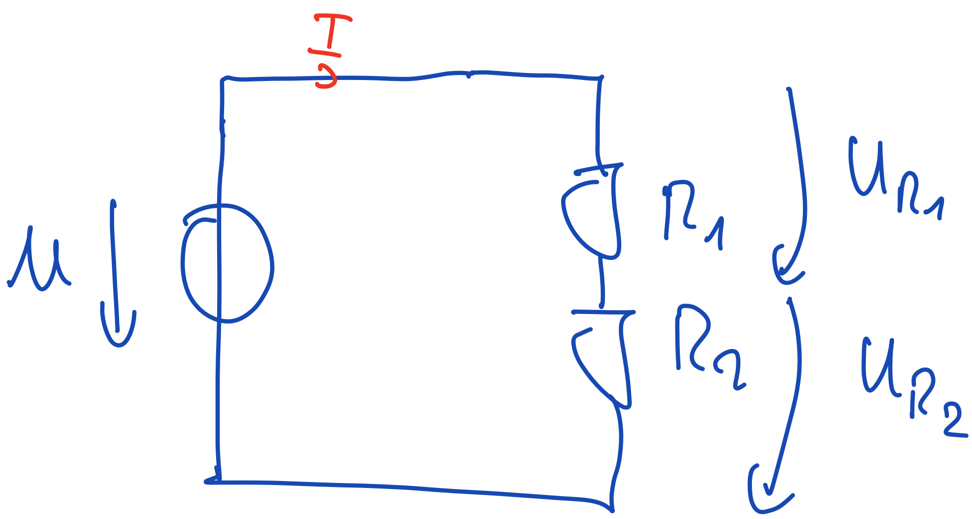
$$I_2 = \frac{U_{R_2}}{R_2} = \frac{U_{R_{eq}}}{R_2}$$

$$I_3 = \frac{U_{R_3}}{R_3} = \frac{U_{R_{eq}}}{R_3}$$

5.4.3 Exemple :



5.5.1 Diviseur de tension :

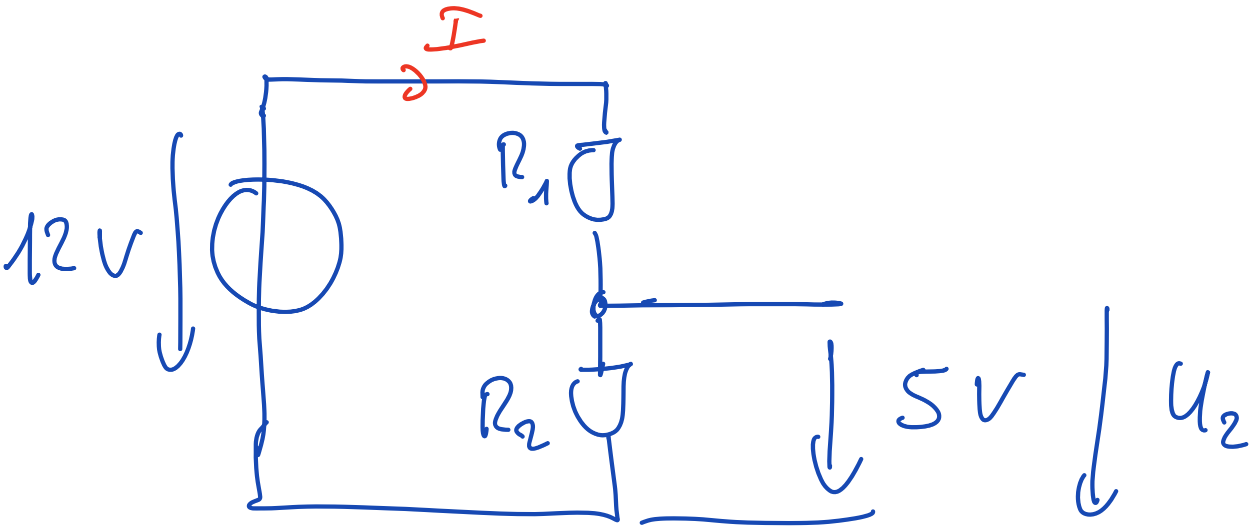


$$U = U_{R_1} + U_{R_2}$$

$$= (R_1 + R_2)\, I$$

$$I = \frac{U}{R_1 + R_2}$$

$$U_{R_2} = R_2 \cdot I = \frac{R_2}{R_1 + R_2} \cdot U$$



$$U_2 = \frac{R_1}{R_1 + R_2} \cdot U$$

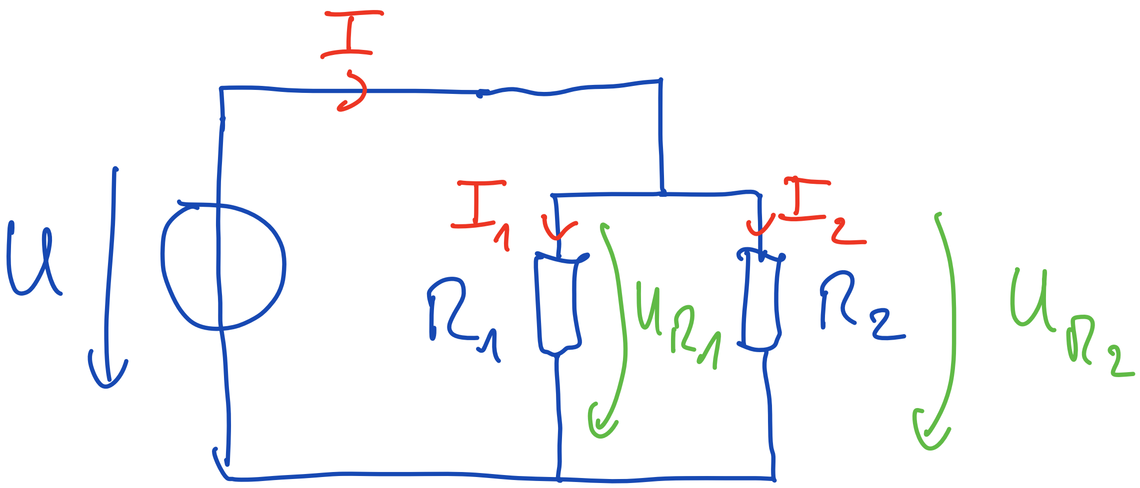
$$5 = \frac{R_2}{R_1 + R_2} \cdot 12$$

$$5 R_1 = 7 R_2$$

$$R_1 = 100\ k\Omega$$

$$R_2 = 71{,}5\ k\Omega$$

5.5.4 Diviseur de courant :



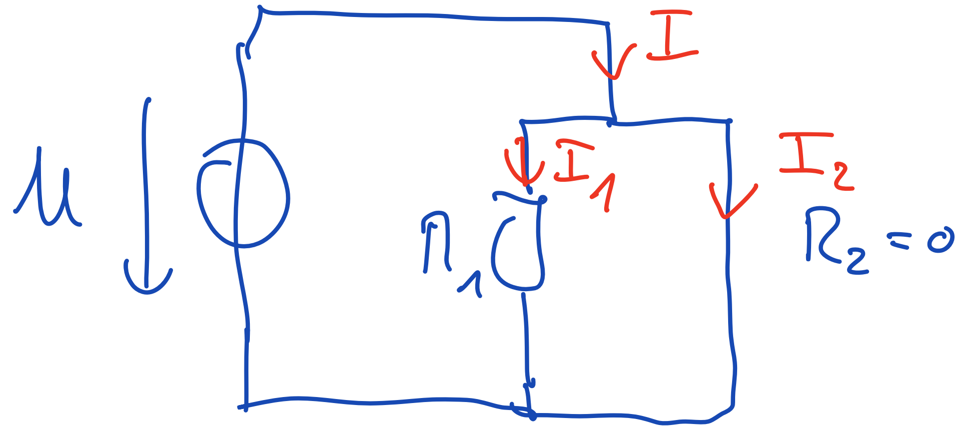
$$U = U_{R_1} = U_{R_2}$$

$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} \qquad I = \frac{U}{R_{eq}}$$

$$U_{R_2} = R_2 \cdot I_2 = U = \frac{R_1 \cdot R_2}{R_1 + R_2} \cdot I$$

$$\boxed{\begin{aligned} I_2 &= \frac{R_1}{R_1 + R_2} \cdot I \\ I_1 &= \frac{R_2}{R_1 + R_2} \cdot I \end{aligned}}$$

Si :



$$I_2 = \frac{R_1}{R_1 + \cancel{R_2}} \cdot I = I$$

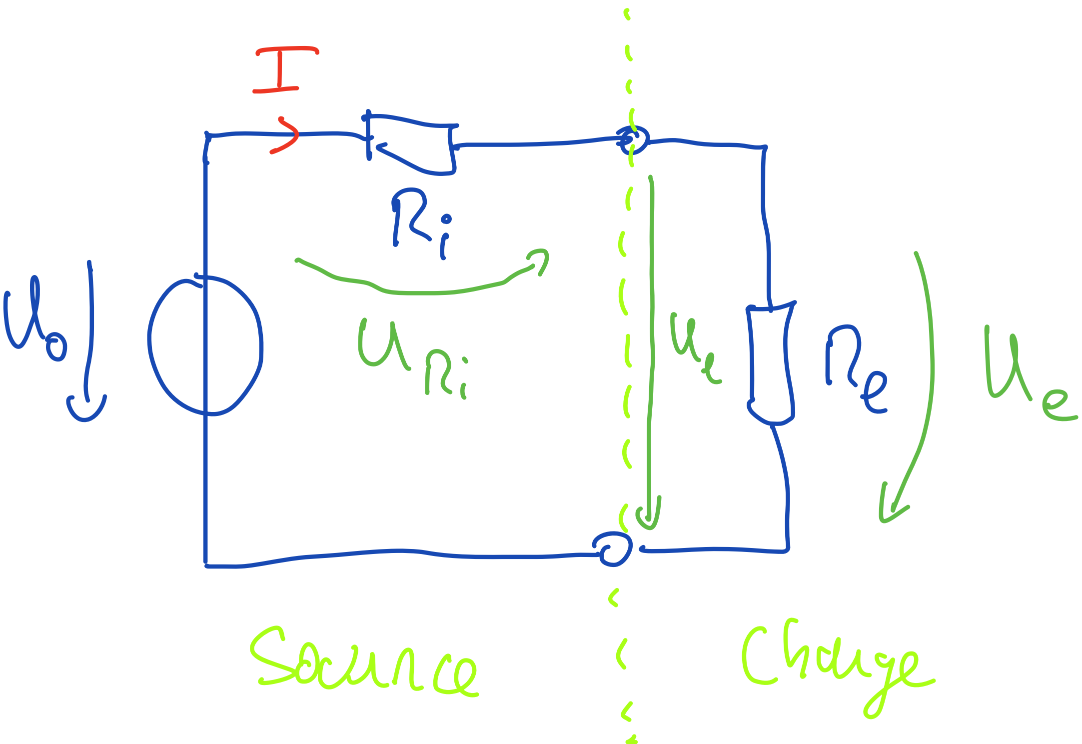
$$I_1 = 0$$

5.6 Méthodes de résolution :

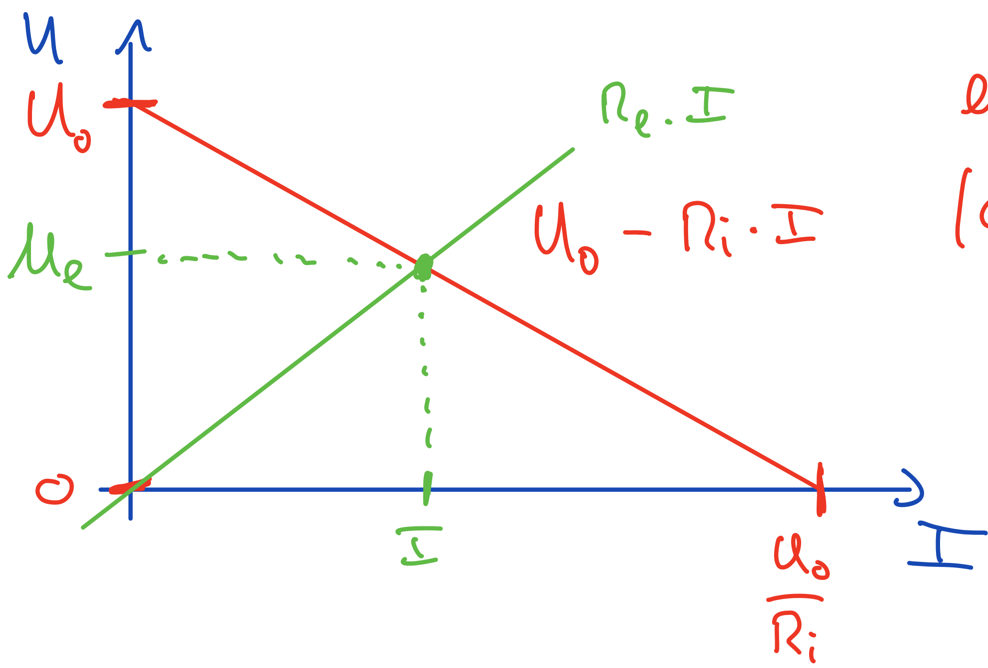
- Redessiner le schéma
- Définir toutes les grandeurs
  U, I, → indice
- Définir le sens des flèches
- Réduire le schéma, série ou //

• Analyse !

5.6.2 Source de tension réelle :



$$U_e = U_0 - U_{R_i} = U_0 - R_i I$$



en court-circuit

(cc) : $U_e = 0$

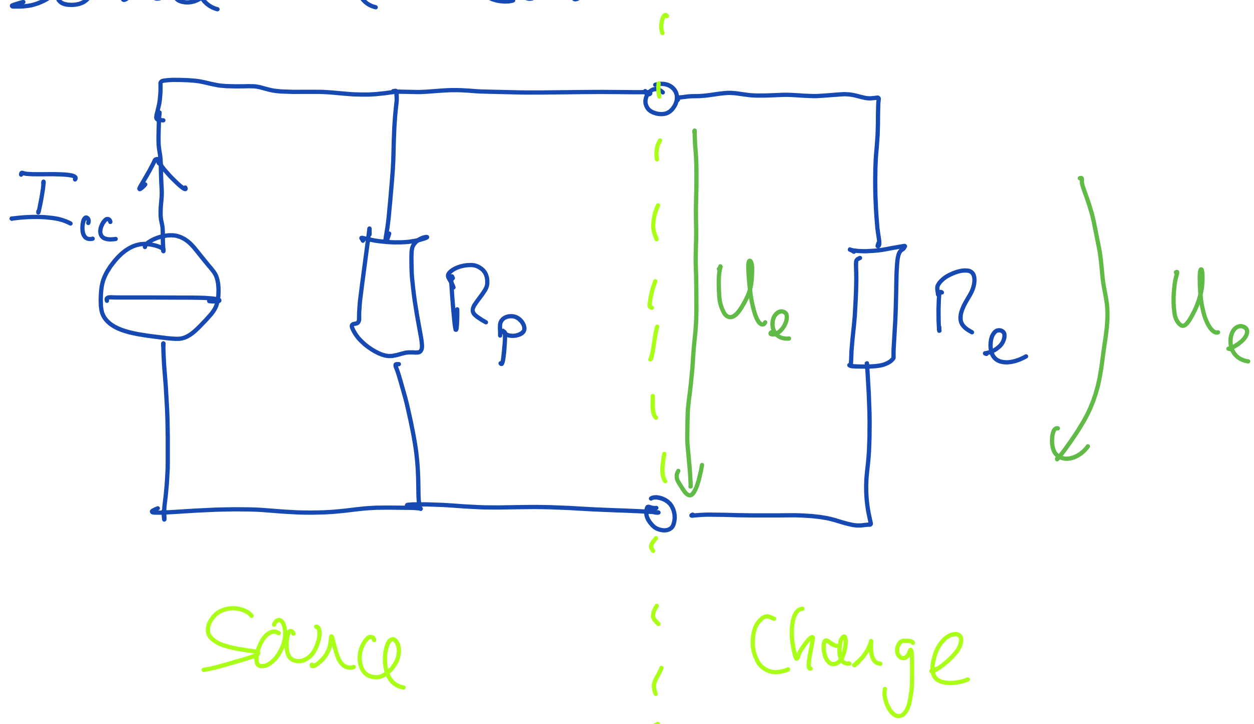
$$I_{cc} = \frac{U_0}{R_i}$$

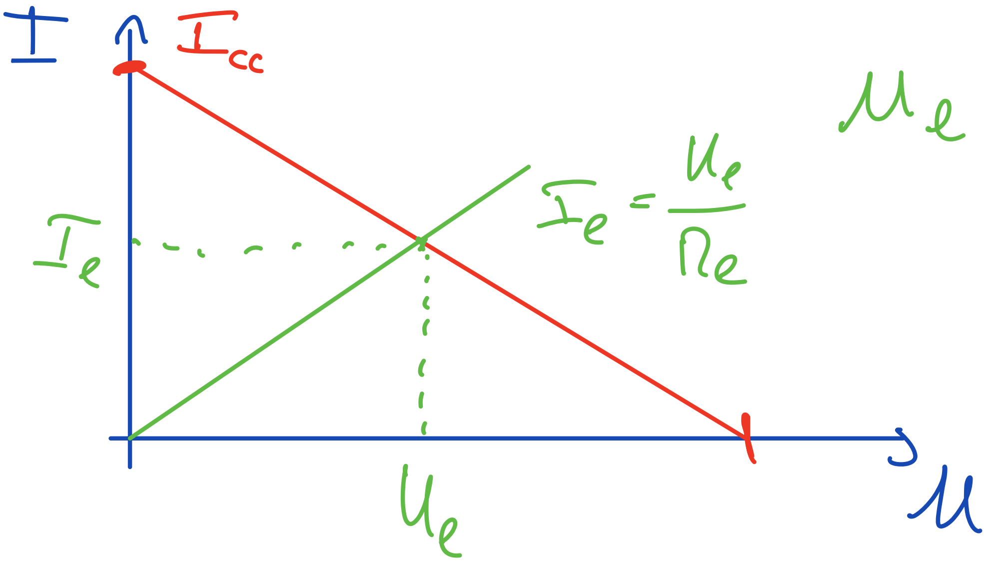
eq de la charge $U_e = R_e \cdot I$

$$U_e = U_0 \cdot \frac{R_e}{R_e + R_i}$$

$$I_e = \frac{U_0}{R_i + R_e}$$

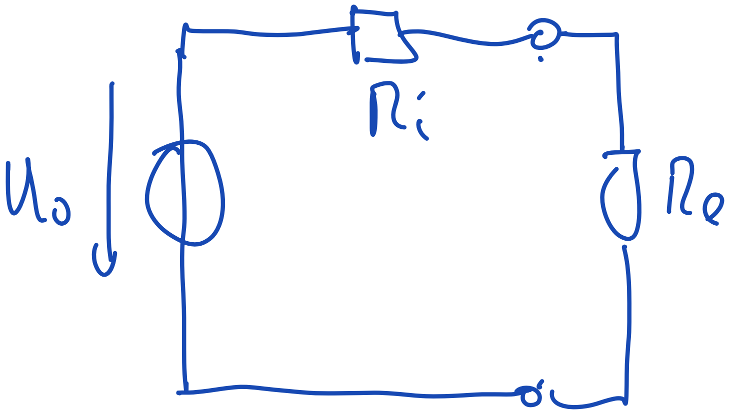
Source de courant réelle :

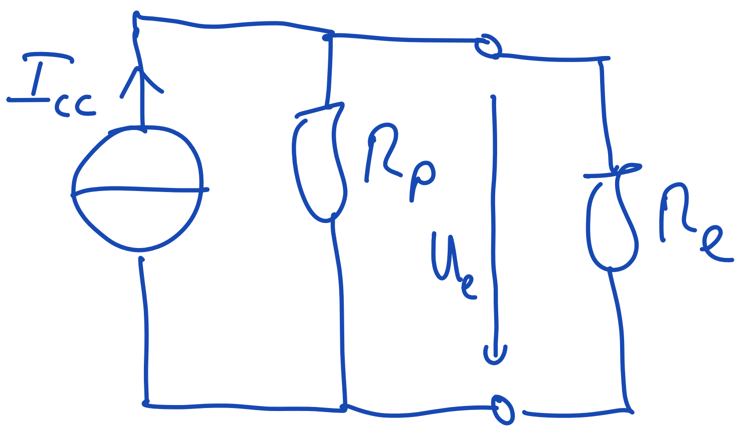




$$U_e = R_e \cdot I_e$$

5.6.3 Equivalence des sources de tension et courant réelles





court-circuit $R_e = 0$	circuit ouvert $R_e \to \infty$
$U_{e_{cc}} = 0$ $I_{e_{cc}} = \dfrac{U_0}{R_i}$	$U_{e_0} = U_0$
$I_{e_{cc}} = I_{cc}$ $U_{e_{cc}} = 0$	$U_{e_0} = R_p \cdot I_{cc}$

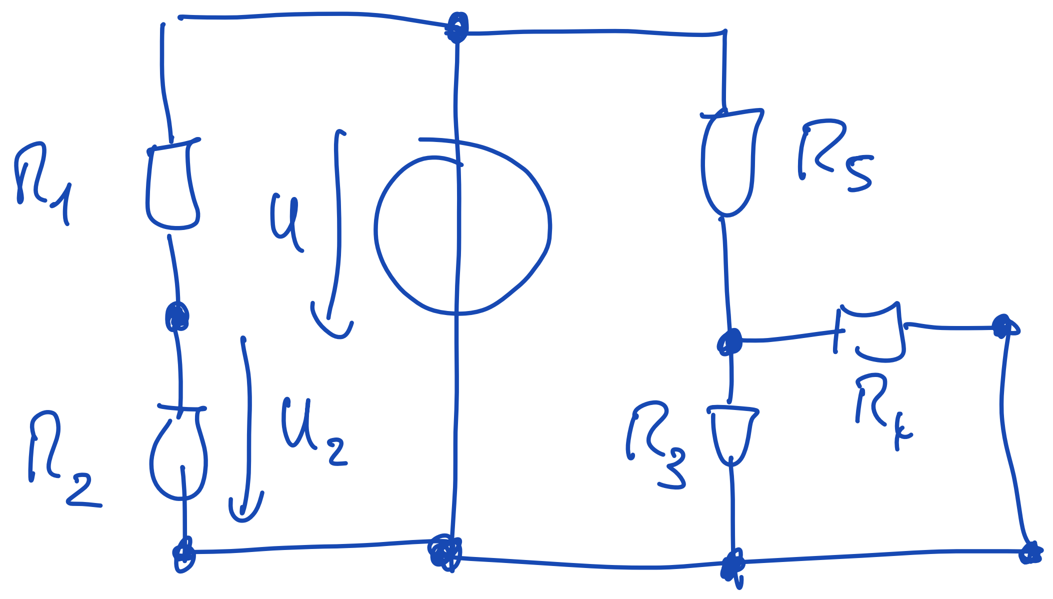
On pose :

$$U_{e_0} = R_p \cdot I_{cc} = U_0$$

$$I_{e_{cc}} = \frac{U_0}{R_i} = I_{cc}$$

$$R_p = \frac{U_0}{I_{cc}} = \frac{U_0}{U_0 / R_i} = R_i$$

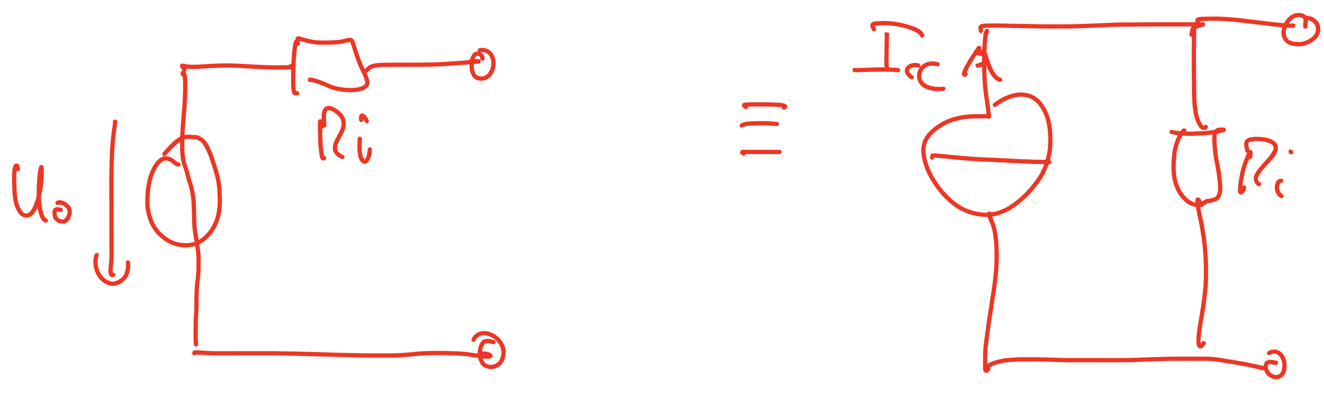
Quiz :



$U = 12\,V$

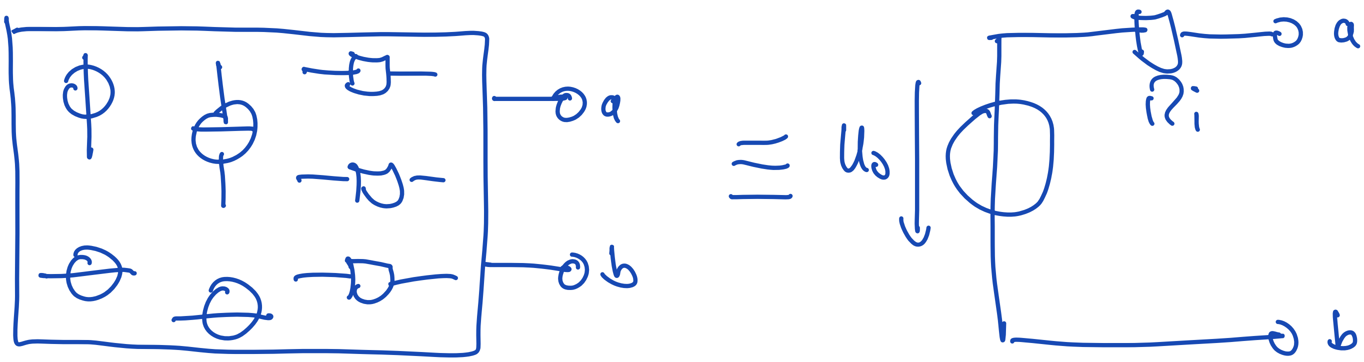
$$U_2 = U \cdot \frac{R_2}{R_1 + R_2} = 5\,V$$

En résumé :



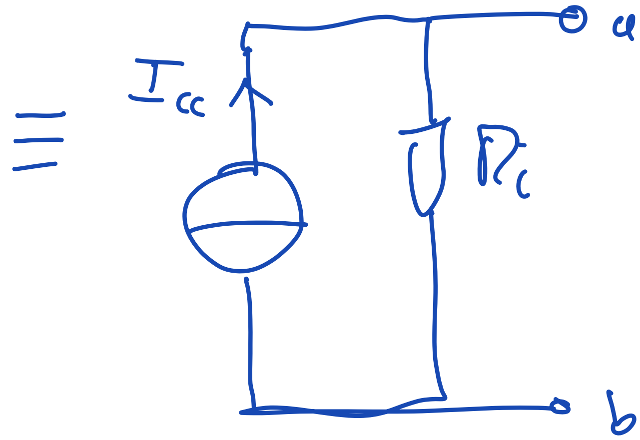
$$I_{cc} = \frac{U_0}{R_i}$$

5.7 Théorèmes de Thévenin et Norton :



U_0 = Tension à vide

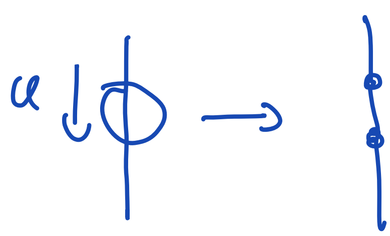
$$U_0 = U_{ab}\Big|_{I_{ab}=0} \quad \text{(à vide)}$$

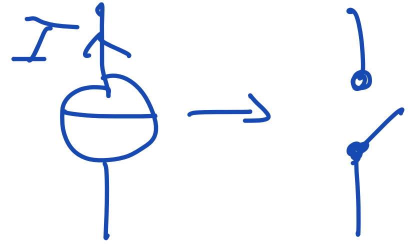


$$I_{cc} = I_{ab}\Big|_{U_{ab}=0} \quad \text{(en court-circuit)}$$

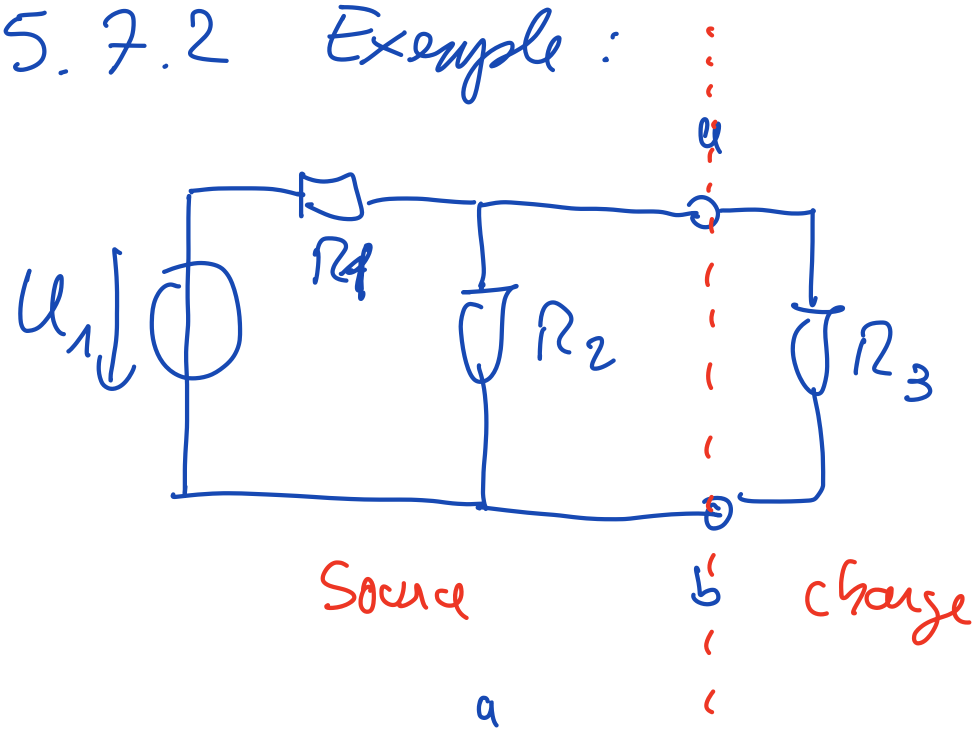
$$R_i = \frac{U_0}{I_{cc}} = R_{ab}\Big|_{\substack{U_j=0 \\ I_j=0}}$$

Annuler une source $\Rightarrow$

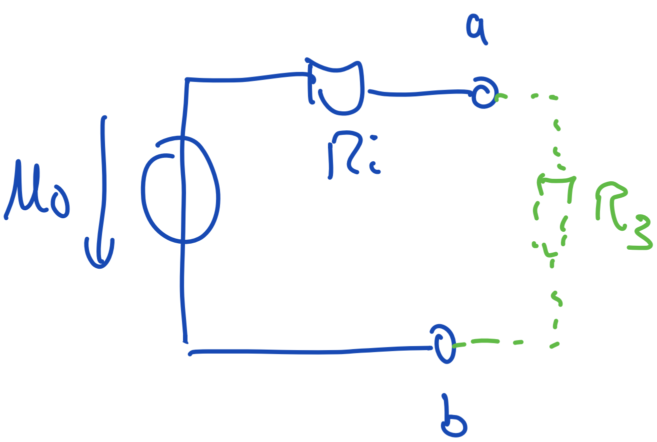




5.7.2 Exemple :

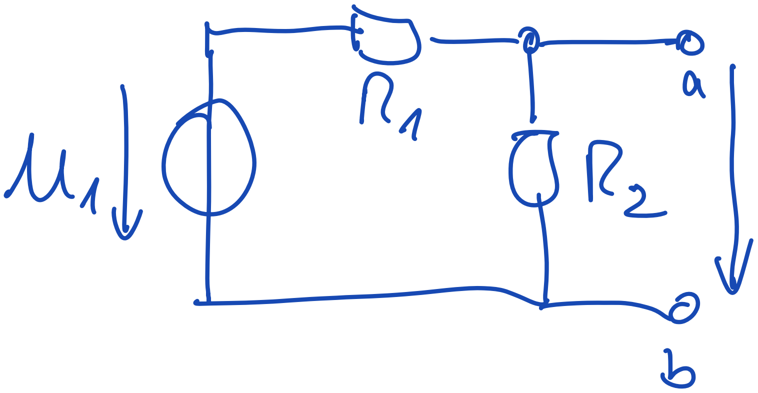


$u_3 = f(R_3)$



u_0 ? Tension à vide entre a et b ?

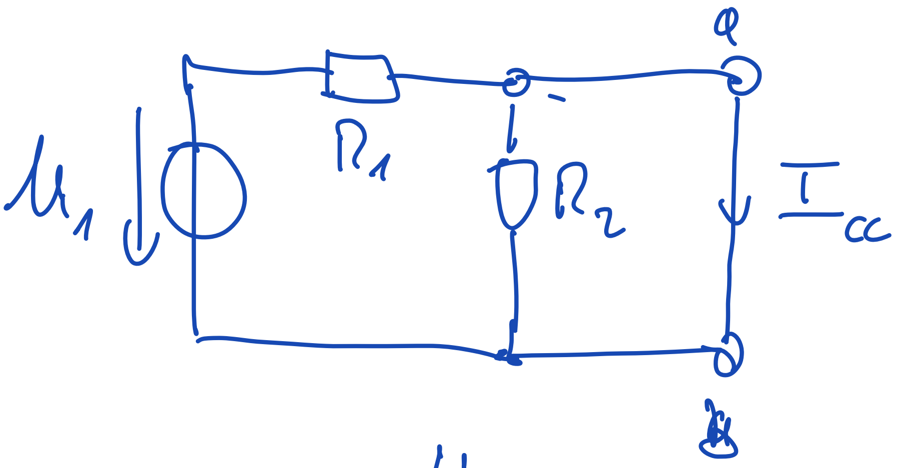
a)



$u_{R_2} = u_0$

$$u_{R_2} = u_0 = u_1 \cdot \frac{R_2}{R_1 + R_2}$$

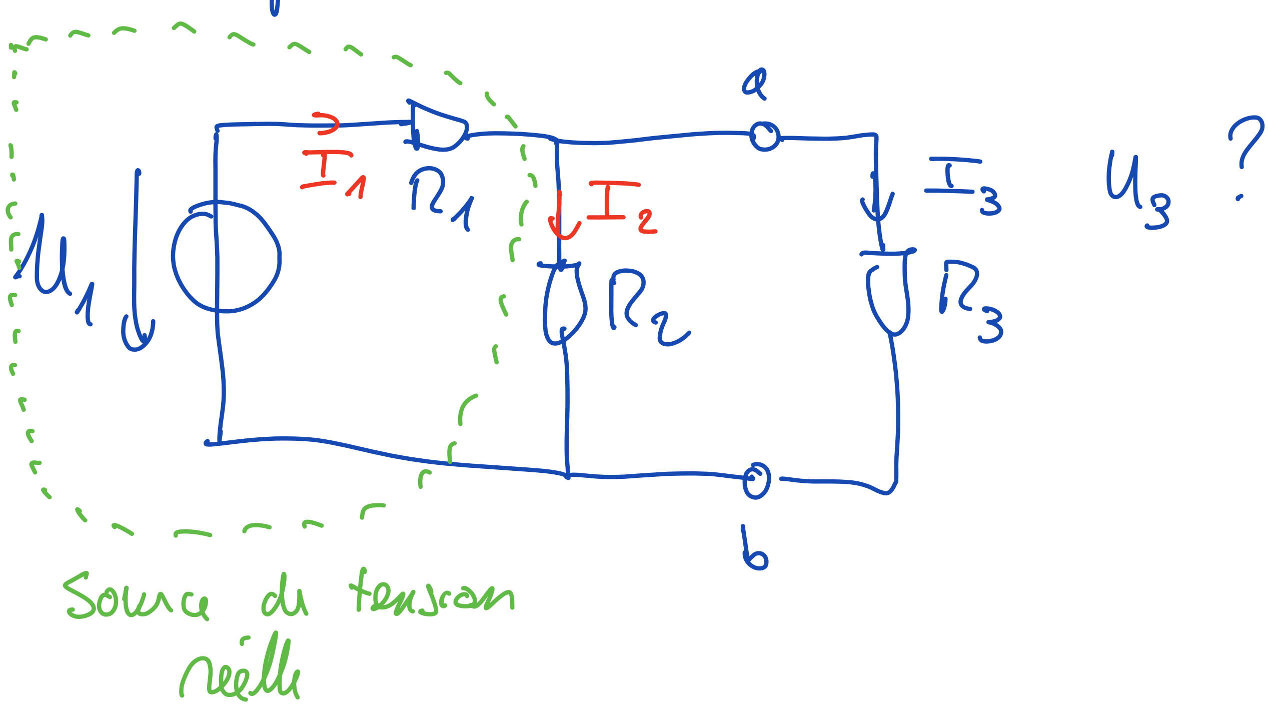
b) I_{cc} = courant de court circuit

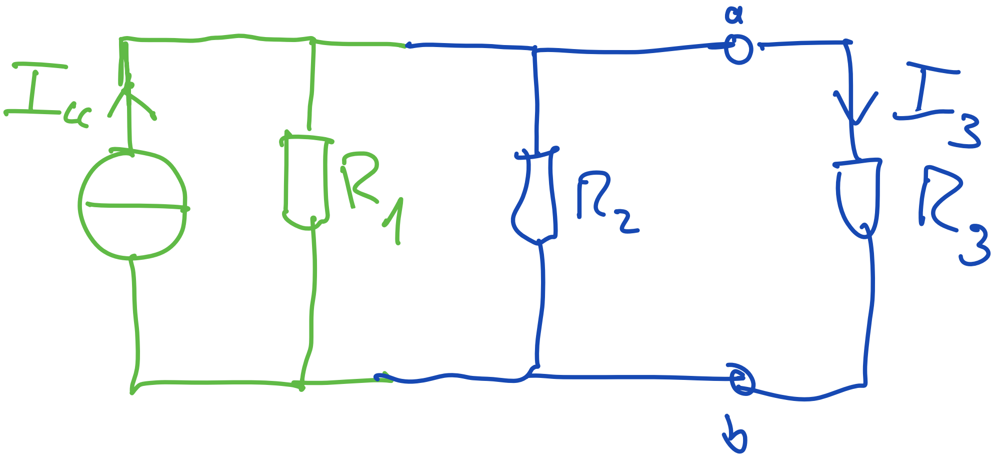


$$I_{cc} = \frac{U_1}{R_1}$$

c) $$R_i = \frac{U_0}{I_{cc}} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

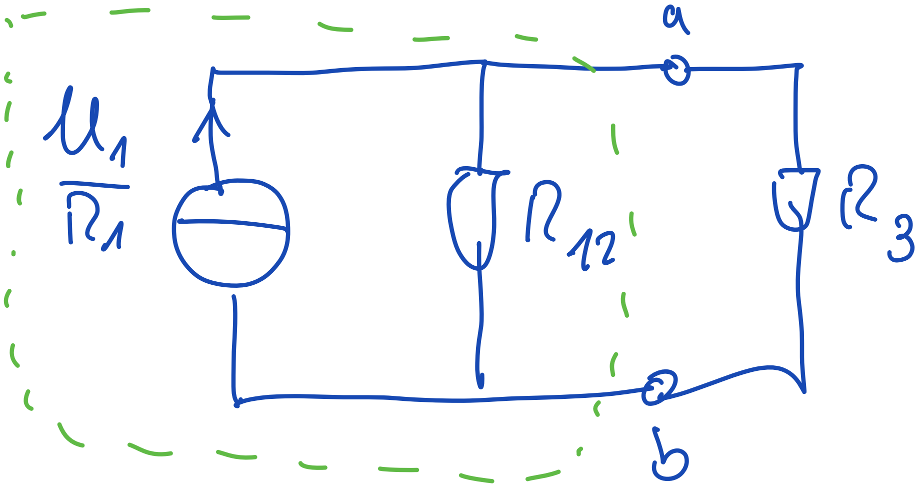
Autre possibilité :



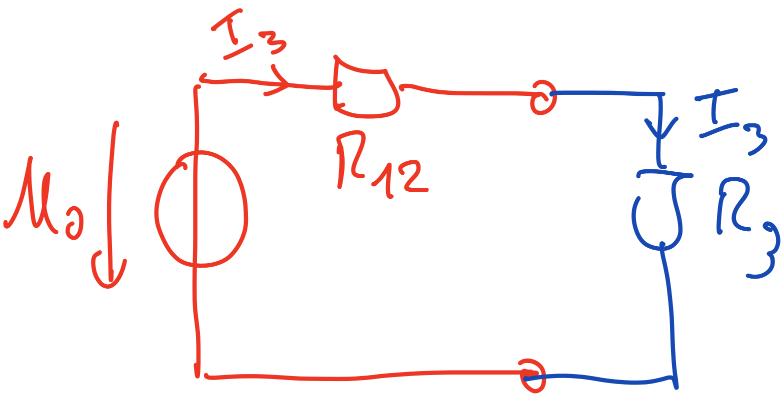


$$I_{cc} = \frac{U_1}{R_1}$$

$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$



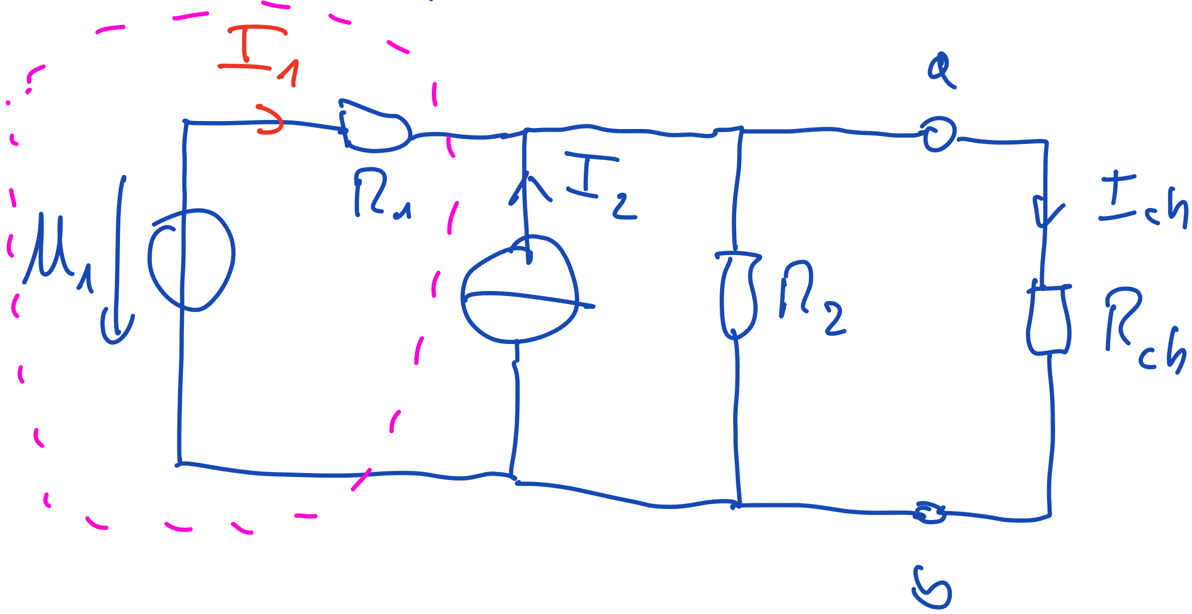
Source de courant réelle

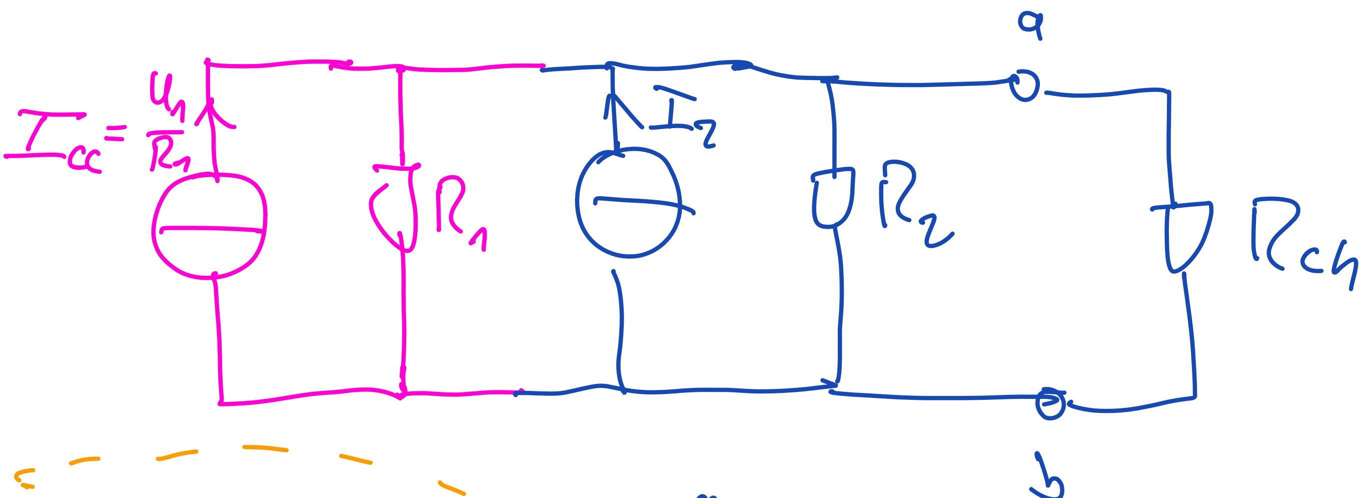


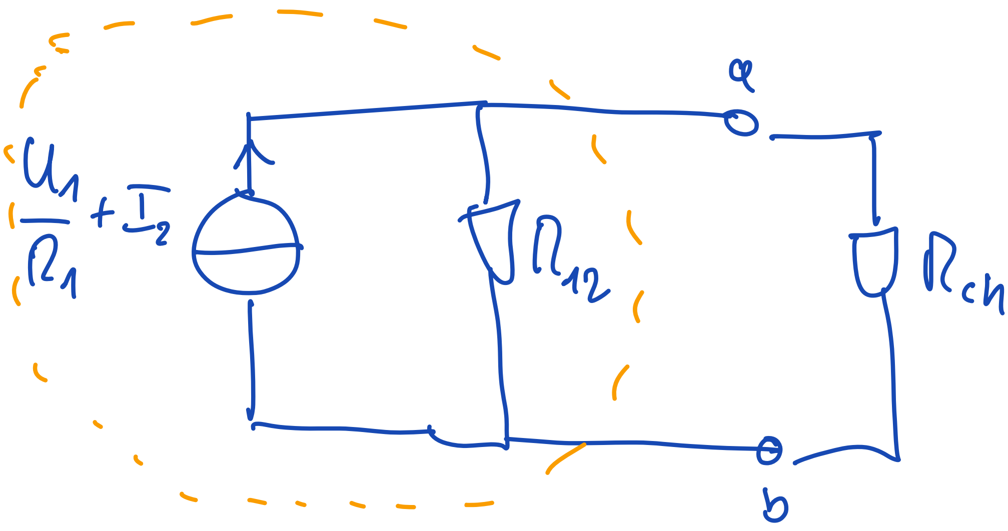
$$U_0 = R_{12} \cdot \frac{U_1}{R_1}$$

$$I_3 = \frac{U_0}{R_{12} + R_3}$$

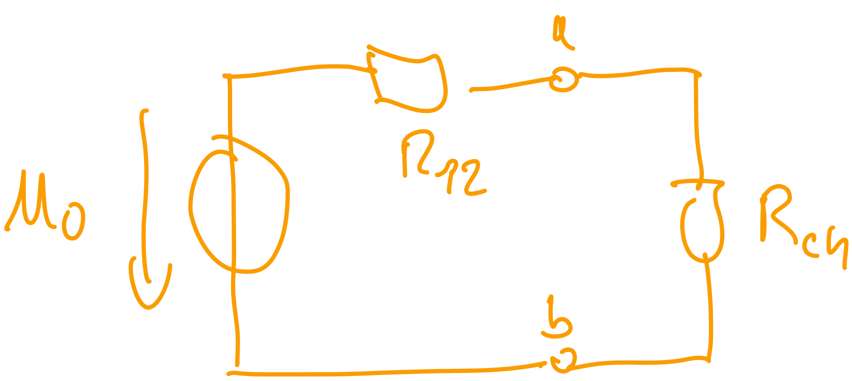
Autre exemple :







$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

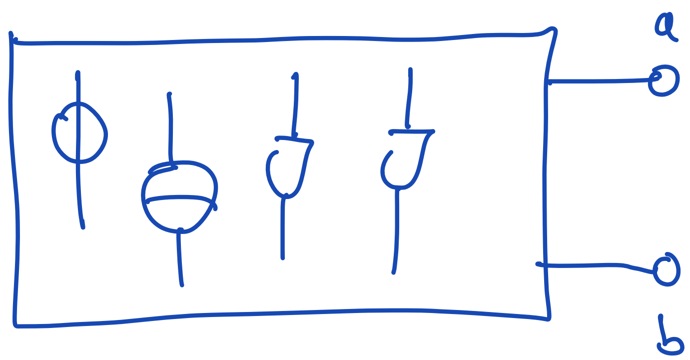


$$U_0 = R_{12}\left(\frac{U_1}{R_1} + I_2\right)$$

$$I_{ch} = \frac{U_0}{R_{12} + R_{ch}}$$

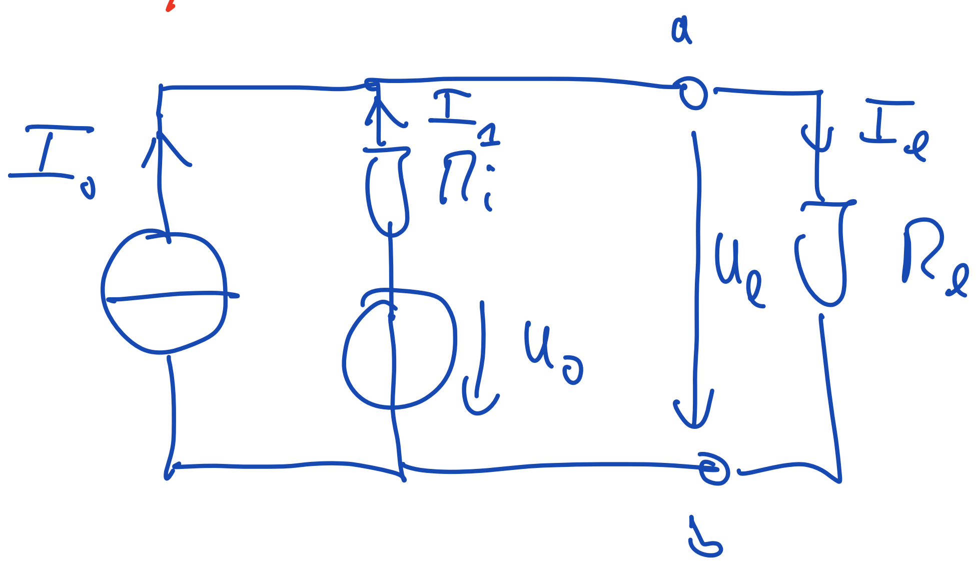
5.8 Principe de superposition

Définition :



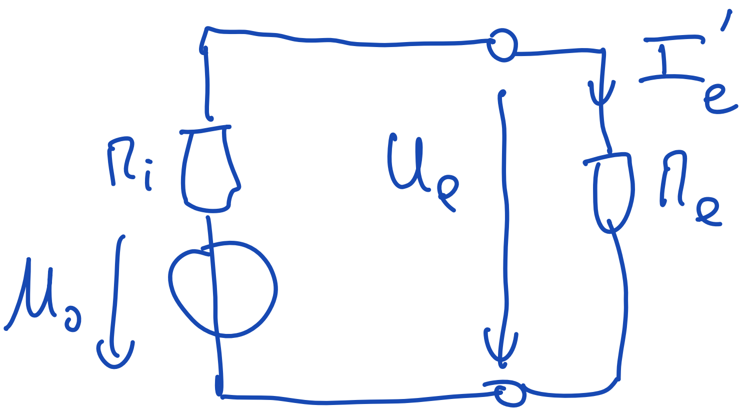
L'action résultante est la somme algébrique des actions séparées de chaque source, les autres étant annulées

Le système doit être linéaire !!



I_e, U_e ?

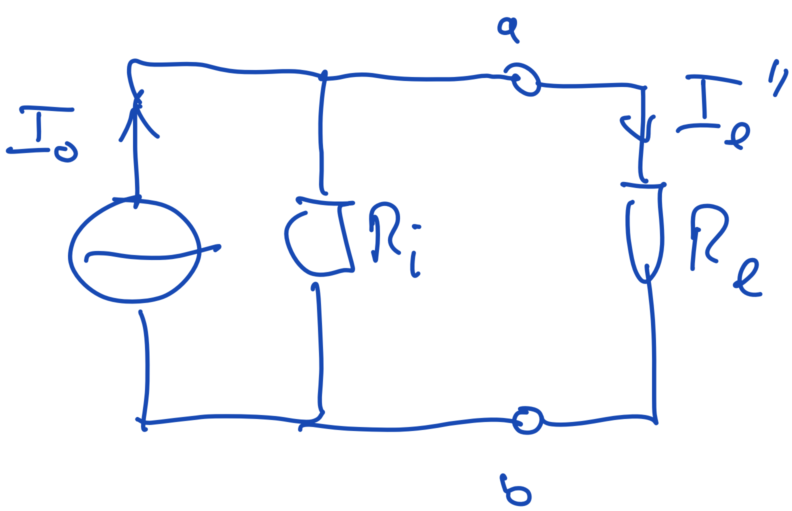
1) On annule la source de courant :



$$I_e' = \frac{U_0}{R_i + R_e}$$

$$U_e' = U_0 \cdot \frac{R_e}{R_e + R_i}$$

2) On annule la source de tension :



$$I_e'' = I_0 \cdot \frac{R_i}{R_i + R_e}$$

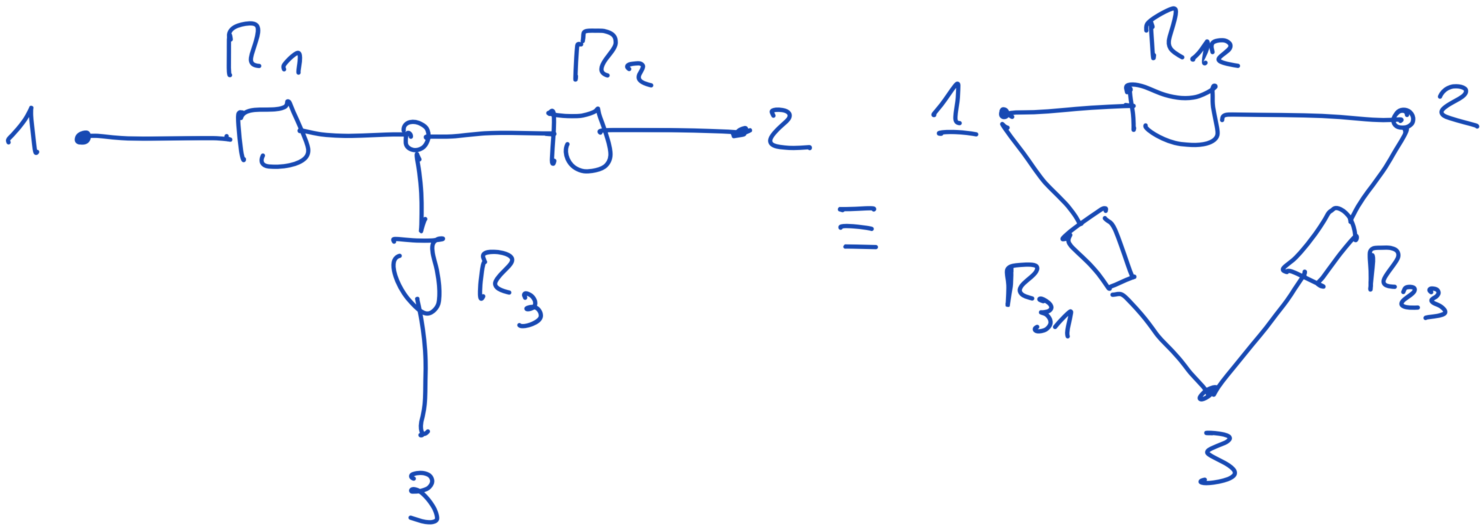
$$U_e'' = R_e \cdot I_e'' = I_0 \cdot \frac{R_e \cdot R_i}{R_e + R_i}$$

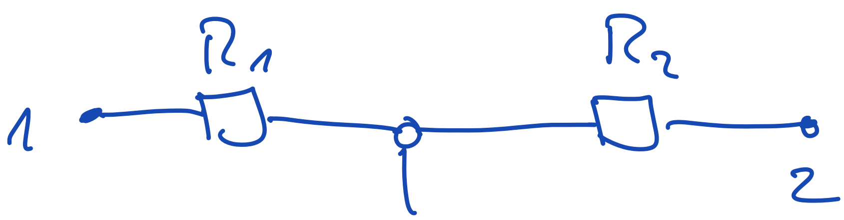
$$I_e = I_e' + I_e''$$

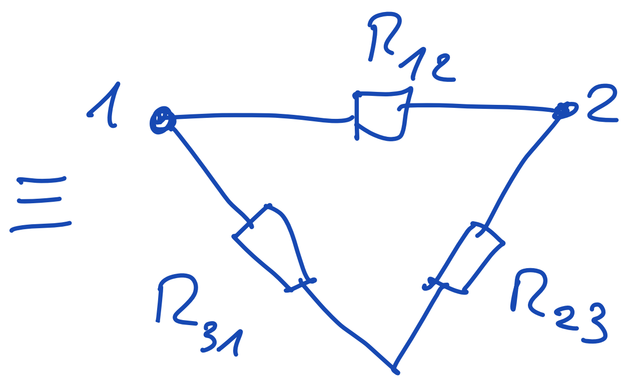
$$u_e = u_e' + u_e''$$

5.9 Transformation Etoile - Triangle

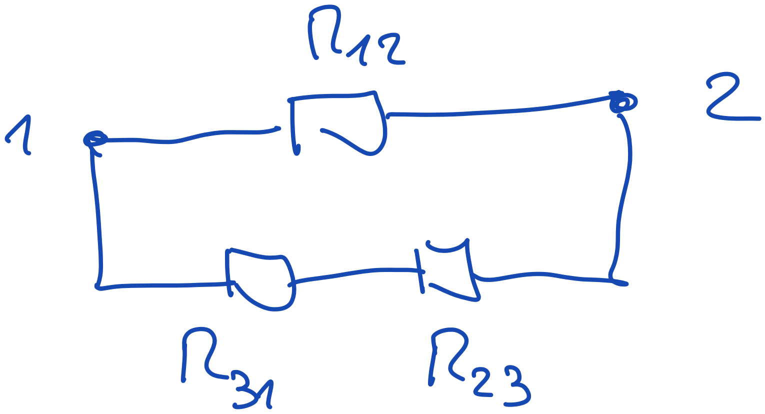
il s'agit d'un tripole :







$$R_{eq} = R_1 + R_2$$



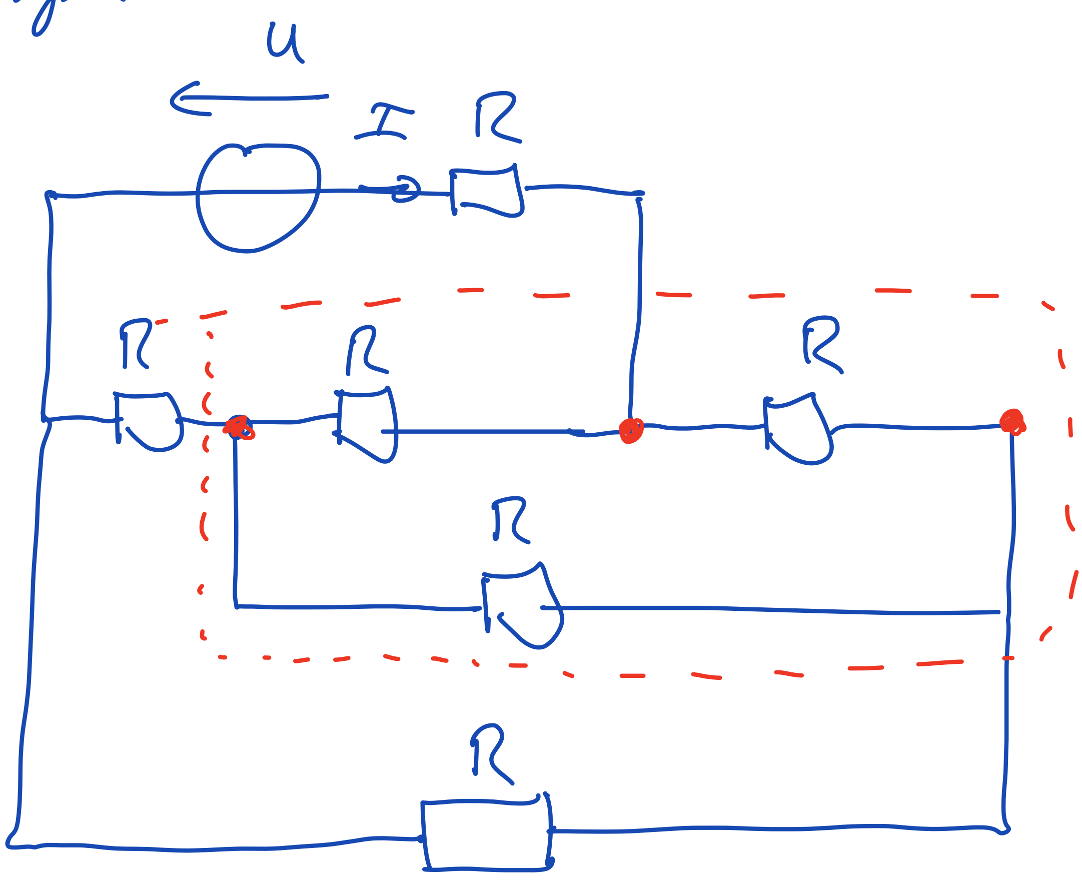
$$R_{eq} = \frac{R_{12} \cdot (R_{31} + R_{23})}{R_{12} + R_{31} + R_{23}}$$

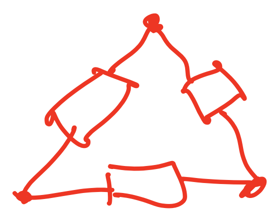
$$R_1 = \frac{R_{31} \cdot R_{12}}{R_{12} + R_{23} + R_{31}} \qquad R_{12} = R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}$$

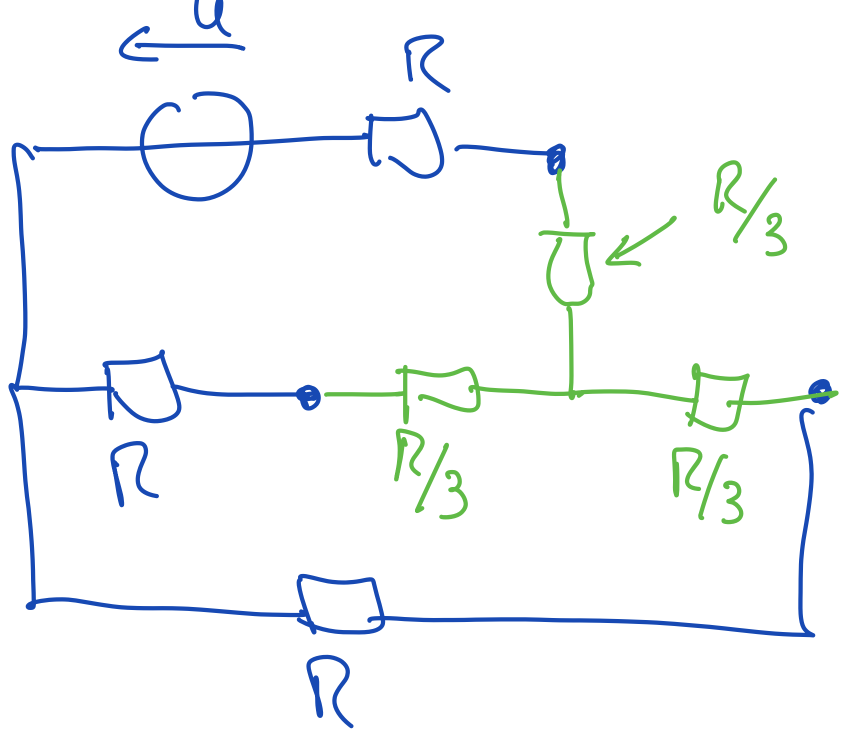
$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{31}} \qquad R_{23} = R_2 + R_3 + \frac{R_2 \cdot R_3}{R_1}$$

$$R_3 = \frac{R_{23} \cdot R_{31}}{R_{12} + R_{23} + R_{31}} \qquad R_{31} = R_3 + R_1 + \frac{R_3 \cdot R_1}{R_2}$$

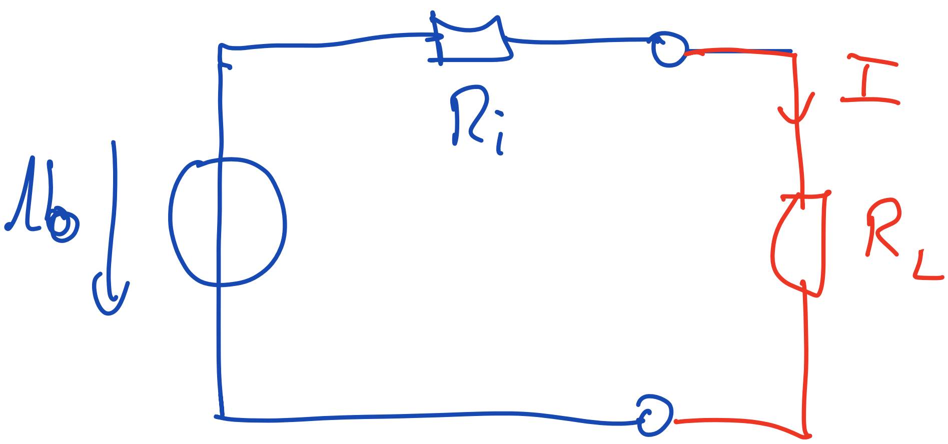
Exemple :







5.11 Puissance maximum transmise par un dipôle :



Puissance transmise à $R_L \rightarrow$ max

$$P_{R_L} = R_L \cdot I^2 = U_{R_L} \cdot I$$

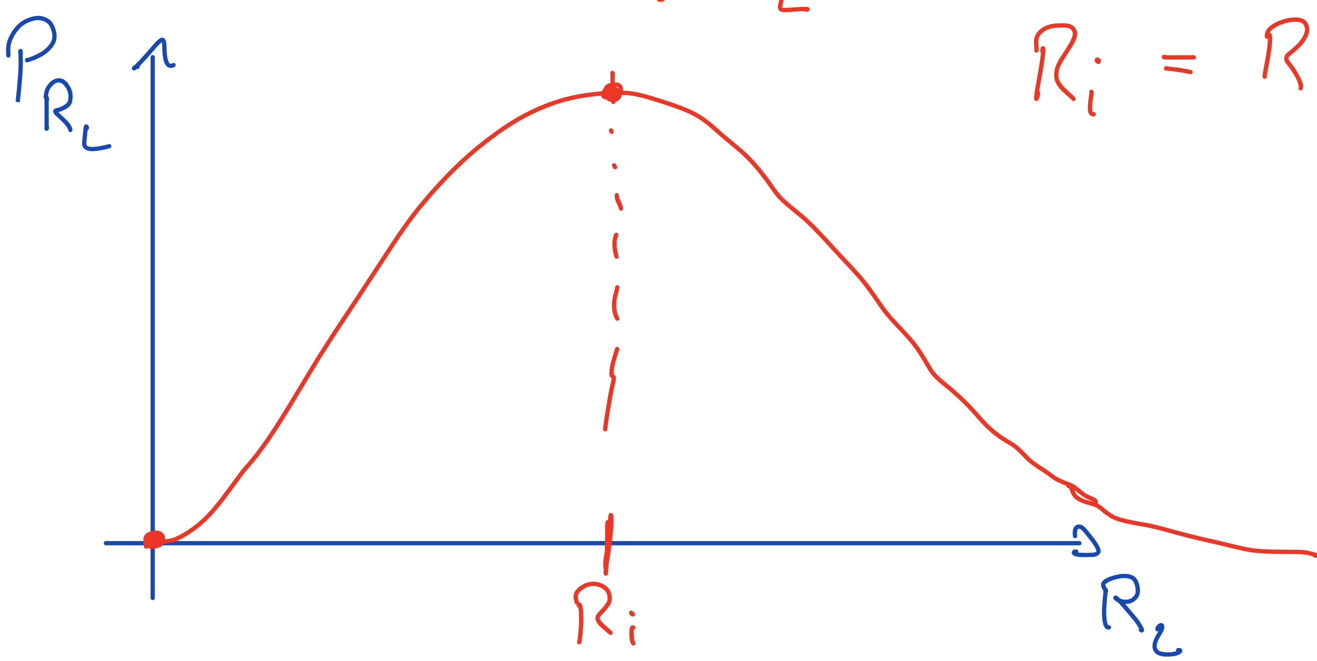
$$I = \frac{U_0}{R_i + R_L}$$

$$P_{R_L} = R_L \frac{U_0^2}{(R_i + R_L)^2}$$

Que doit valoir R_L par rapport à R_i pour P_{R_L} max

Max : $\rightarrow$ $$\frac{d P_{R_L}}{d R_L} = 0$$

$R_i = R_L$



$\rightarrow$ l'adaptation de puissance

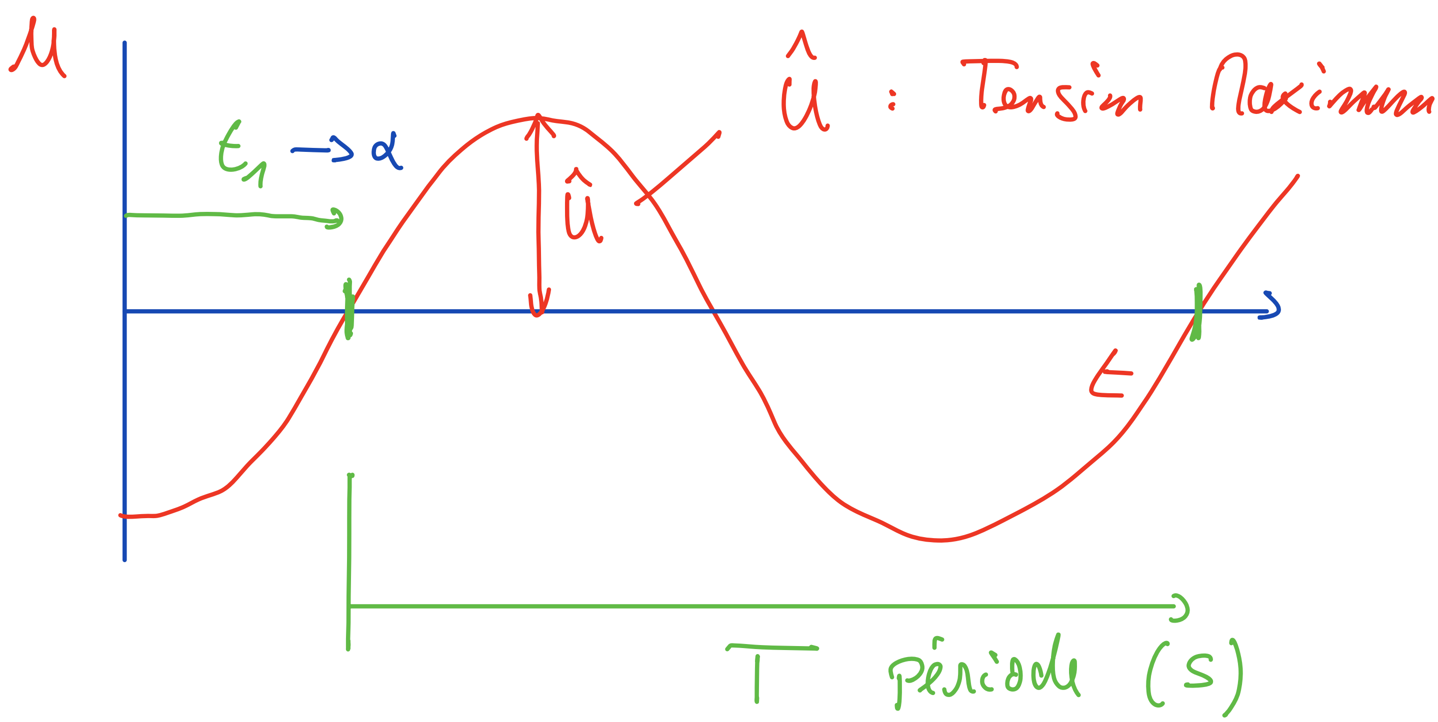
$$\eta = \frac{P_{utile}}{P_{consommée}} = \frac{R_L \cdot \cancel{I^2}}{(R_L + R_i)\cancel{I^2}}$$

Si $R_i = R_L$

$\Rightarrow \quad \eta = 0{,}5$

6. Régime sinusoïdal Monophasé :

6.2 Grandeurs Sinusoïdales



T = période du signal (s)

f = fréquence $= \dfrac{1}{T}$ (Hz)

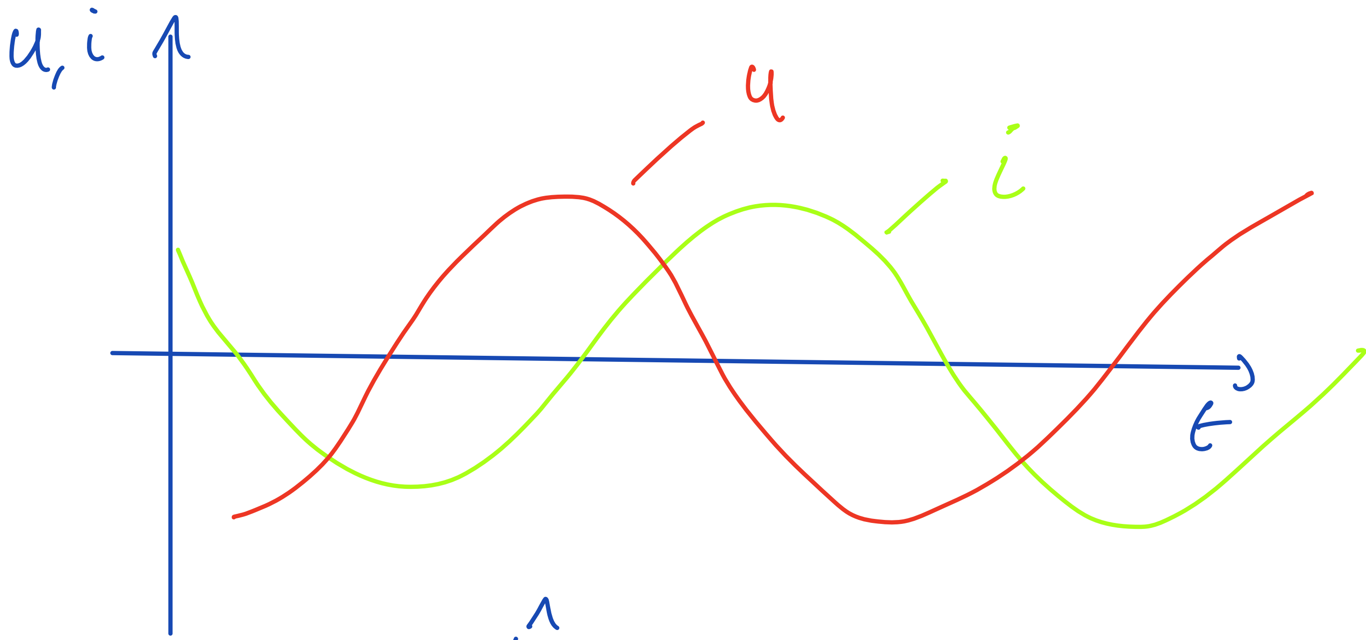
ω = pulsation $= \dfrac{2\pi}{T} = 2\pi \cdot f$

$$\alpha = \frac{t_1 \cdot 2\pi}{T}$$

$$u(t) = \hat{U} \sin(\omega t + \alpha)$$

$$u = \hat{U} \sin(\omega t + \alpha$$

minuscule : → grandeur instantanée



$$u = \hat{U} \sin(\omega t + \alpha)$$

$$i = \hat{I} \sin(\omega t + \beta)$$

Définition : $\varphi = \alpha - \beta$

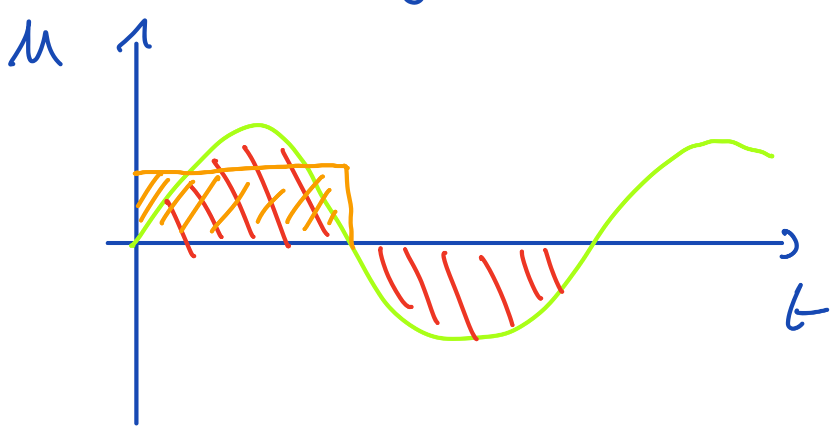
déphasage entre u et i

Définition de la valeur moyenne :

$$\overline{x} = \frac{1}{T}\int_0^T x(t)\,dt$$

$$\overline{u} = \frac{1}{T}\int_0^T u\,dt$$

$$\overline{u} = \frac{1}{T}\int_0^T \hat{U}\sin(\omega t+\alpha)\,dt$$



Moyenne sur $T/2$:

$T/2$

$$\bar{u}\Big|_{T/2} = \frac{1}{T/2}\int_0^{T/2} \hat{u}\sin(\omega t)\,dt \qquad \alpha = 0$$

$$\bar{u}\Big|_{T/2} = \frac{1}{\omega}\,\frac{2\hat{u}}{T}\left[-\cos\left(\omega\cdot\frac{T}{2}\right) + \cos(0)\right]$$

$$\omega = \frac{2\pi}{T}$$

$$\frac{\cancel{2}\,\hat{u}\,\cancel{T}}{\cancel{T}\,\cancel{2}\cdot\pi}\left[-\cos\left(\frac{\cancel{T}}{\cancel{2}}\,\frac{\cancel{2}\pi}{\cancel{T}}\right) + \cos(0)\right]$$

$$= \frac{\hat{u}}{\pi}\Big[1 + 1\Big] = \frac{2}{\pi}\hat{u}$$

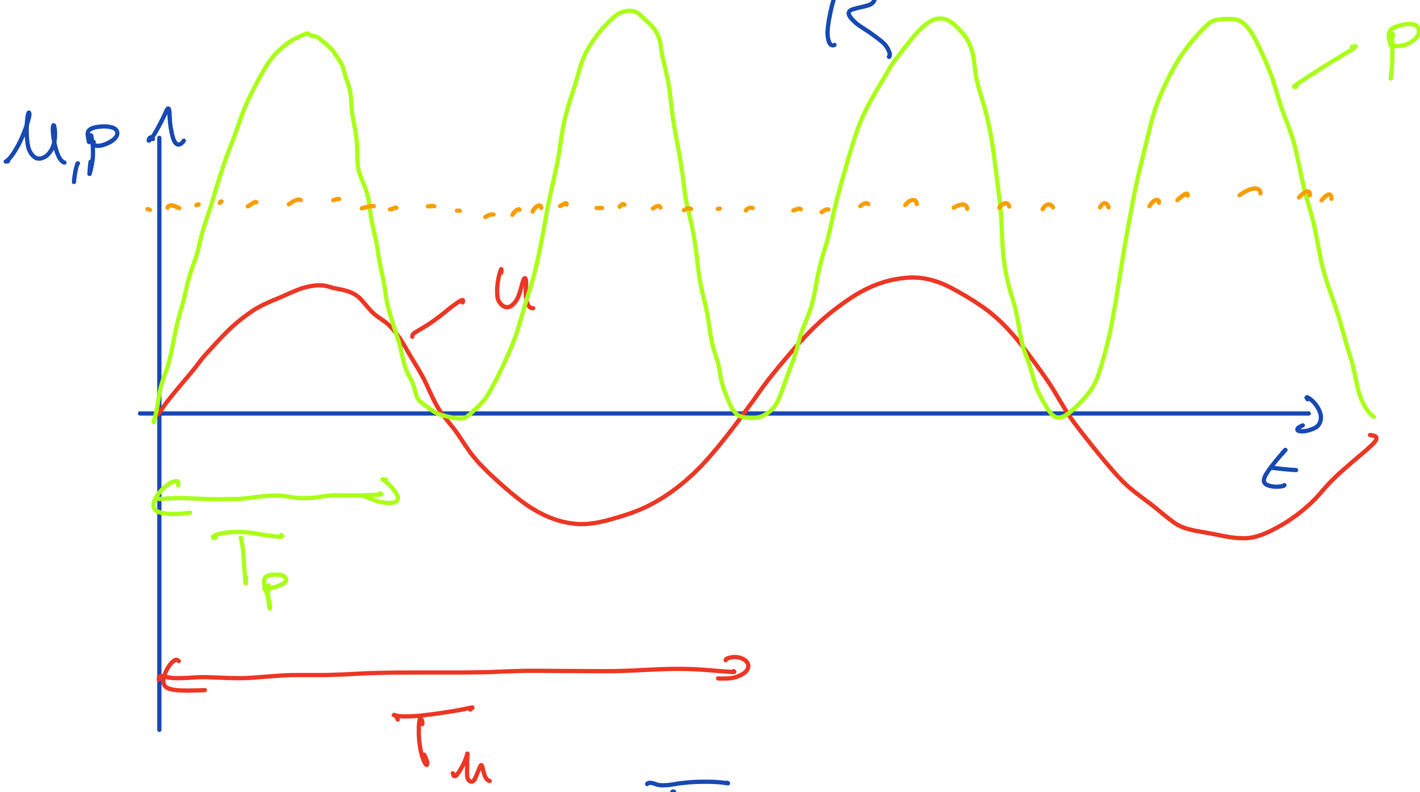
6.2.13 Puissance instantanée

$$p = u \cdot i$$

$$p = R \cdot i^2 = \frac{u^2}{R}$$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$$= \frac{\hat{U}^2 \sin^2(\omega t + \alpha)}{R}$$



$$\bar{P}_R = \frac{1}{T} \int_0^T \frac{\hat{U}^2}{R} \sin^2(\omega t + \alpha)\, dt$$

$$\bar{P}_R = \frac{1}{T} \int_0^T \frac{\hat{U}^2}{R} \cos^2(\omega t + \alpha)\, dt$$

$$2\,\overline{P}_R = \frac{1}{T}\int_0^T \frac{\hat{u}^2}{R}\cdot \underline{1}\; dt$$

$$2\,\overline{P}_R = \frac{\hat{u}^2}{R} \rightarrow \overline{P}_R = \frac{\hat{u}^2}{2R}$$

En alternatif : $\overline{P}_R = \dfrac{\hat{u}^2}{2R}$

En continu : $P_R = \dfrac{u^2}{R}$

6.2.12 Valeur efficace :

(RMS Root Mean. Square)

Déf : $U = \sqrt{\dfrac{1}{T}\displaystyle\int_0^T \hat{u}^2 \sin^2(\omega t+\alpha)\, dt}$

Majuscule

$\sin^2 x$

$$= \frac{1-\cos 2\alpha}{2}$$

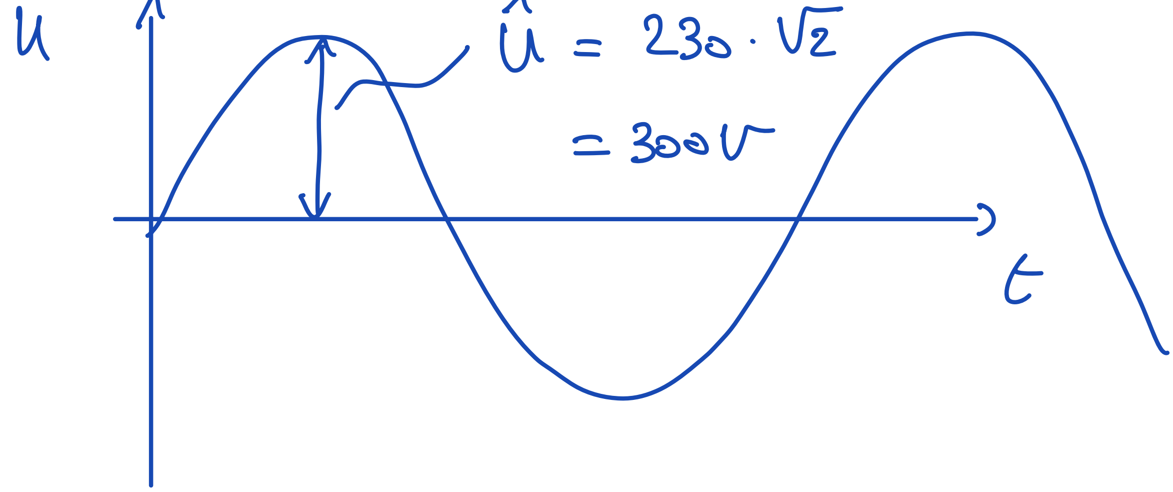
$$= \sqrt{\frac{\hat{u}^2}{T}\int_0^T \frac{1}{2}\,dt - \frac{1}{2}\frac{\hat{u}^2}{T}\underbrace{\int_0^T \cos(2\omega t - 2\alpha)\,dt}_{0}}$$

$$\boxed{u = \frac{\hat{u}}{\sqrt{2}}}$$

$$\hat{u} = u \cdot \sqrt{2}$$

crête — efficace — coef. avec sinus

$$\bar{P}_R = \frac{\hat{u}^2}{2R} = \frac{u^2}{R}$$



u, i valeurs instantanées

U, I valeurs efficaces

$\hat{U}, \hat{I}$ valeurs crêtes

$\bar{u}, \bar{i}$ valeurs moyennes

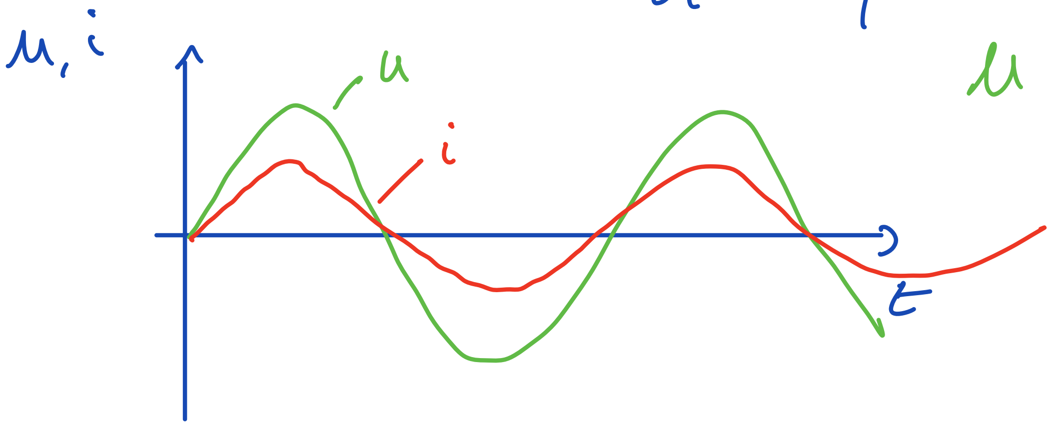
6.2.14 Cas de R

$$u = R \cdot i$$

$$\hat{U} \cos(\omega t + \alpha) = R \cdot \hat{I} \cos(\omega t + \beta)$$

Donc : $\hat{U} = R \cdot \hat{I}$

$$\alpha = \beta$$



u est en phase avec i

6.2.15 cas de L

$$u = L \frac{di}{dt}$$

$$\hat{U} \cos(\omega t + \alpha) = -\omega L \hat{I} \sin(\omega t + \beta)$$

$$= \omega L \hat{I} \cos\left(\omega t + \beta + \frac{\pi}{2}\right)$$

$$\hat{U} = \omega L \hat{I}$$

$$\alpha = \beta + \frac{\pi}{2}$$

Tension et le courant sont quadrature

retard du courant de $\frac{\pi}{2}$ sur la tension.

6.2.16 cas de C

$$i = C \frac{du}{dt}$$

$$\hat{I} \cos(\omega t + \beta) = -\omega C \hat{U} \sin(\omega t + \alpha)$$

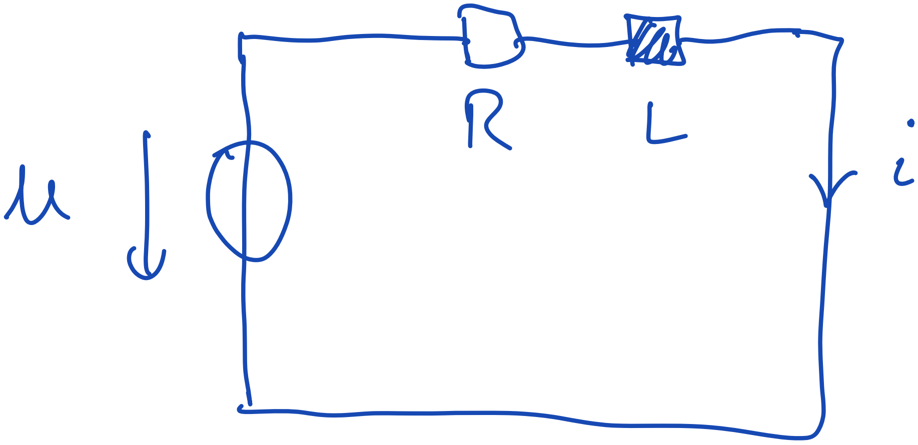
$$= \omega C \hat{U} \cos\left(\omega t + \alpha + \frac{\pi}{2}\right)$$

$$\hat{U} = \frac{\hat{I}}{\omega C}$$

$$\alpha = \beta - \frac{\pi}{2}$$

courant en avance de $\pi/2$ sur la tension

6.3 Calcul complexe associé :



$u = u_R + u_L$

$u = \hat{U} \sin(\omega t + \alpha)$ connu

$i = \hat{I} \sin(\omega t + \beta)$ inconnu

$\longrightarrow$ simplification : $\alpha = 0$

$\hat{U} \sin(\omega t) = R \cdot \hat{I} \sin(\omega t + \beta) +$

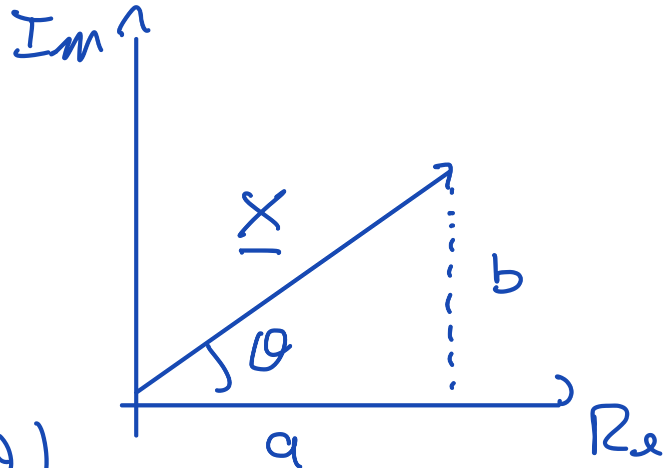
$\omega L \hat{I} \cos(\omega t + \beta)$

⋮

compliqué !!

Rappel : $j = \sqrt{-1}$

$$\underline{X} = a + bj$$



$$= \hat{X}\left(\cos\theta + j\sin\theta\right)$$

$$= \hat{X} e^{j\theta}$$

Concept :

$$u = \hat{U}\sin(\omega t) \xrightarrow{\text{Trans. compl.}} \underline{u} = \hat{U} e^{j(\omega t)}$$

Reel   imaginaire

$$\xleftarrow{u = \mathrm{Im}\{\underline{u}\}}$$

$$i = \hat{I}\sin(\omega t + \beta) \longrightarrow \underline{i} = \hat{I} e^{j(\omega t + \beta)}$$

$$\longleftarrow$$

$$i = \mathrm{Im}\{\underline{i}\}$$